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The Relevance of Extreme Value Theory for Operational Risk Measurement[1]

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Introduction

Operational risk has become an area of growing concern in banking. The increase in the sophistication and complexity of banking practices has raised both regulatory and industry awareness of the need for an effective operational risk management and measurement system. From the time of the release of the second consultative document on the New Capital Accord, the Basel Committee on Banking Supervision has established a specific treatment for operational risk: a basic component of the new framework is represented by Pillar 1, which explicitly calls for a minimum capital charge for this category of risk.

The capital charges on operational risk are basically adopted the current methods developed by BIS. However, these methods could not recognize the distribution of skewed data. In this paper, we suggest that extreme value theory is more relevant to measuring operational risk. This theory is much more critical about internal statistical models for operational risk.

The remaining discussion will be divided into three sections. In the following section, the basic definition of operational risk will be discussed. Then, the current methods for calculating operational risk will be highlighted. Finally, the relevance of extreme value theory for measurement of operational risk is suggested.

Some Background on Operational Risk

What is Operational Risk?

There is no definition that is an "acceptable and recognized" explanation for operational risk, as this is yet to evolve. However, we can describe operational risk generally as follow. It ranges from narrow definition of covering operational breakdowns in processes to broad definitions, which capture all risks that are not credit or market risks.

For banking industries, the *Basle Committee* has adopted a common industry definition of operational risk, *can be described as the "risk of loss, resulting from inadequate or failed internal processes, people and systems, or from external events."[4]* . Possible operational risk categories, as suggested by van den Brink (2002) are:

i. Human processing errors, for example, mishandling of software applications, reports containing incomplete information, or payments made to incorrect parties without recovery;

ii. Human decision errors, for example, unnecessary rejection of a profitable trade or wrong trading strategy due to incomplete information;

iii. (Software or hardware) system errors, for example, data delivery or data import is not executed and the software system performs calculations and generates reports based on incomplete data;

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[4] see Basel Committee on Banking Supervision, 2003a,b

iv. Process design error, for example, workflows with ambiguously defined process steps;

v. Fraud and theft, for example, unauthorised actions or credit card fraud;

vi. External damages, for example, fire or earthquake.

This definition includes legal risk, which is the risk of loss resulting from failure to comply with laws as well as prudent ethical standards and contractual obligations. It also includes the exposure to litigation from all aspects of an institution's activities. The definition does *not* include strategic or reputation risks.

Operational Risk Loss

In this section a new term, "*operational risk loss*", will be introduced and discussed. What is an operational risk loss? Operational risk loss is defined as the risk of losses resulting from inadequate or failed internal controls involving processes, people and systems or from external events, which includes but is not limited to legal risk and compliance risk. Operational risk loss also arises due to the failures in governance, business strategy and process. Negative publicity about the Islamic bank's business practices, particularly relating to Shariah non-compliance in their products and services, could have an impact upon their market position, profitability and liquidity.

Legal risk arises from the potential that unenforceable contracts, lawsuits, or adverse judgments can disrupt or otherwise negatively affect the operations or condition of an Islamic bank.

Therefore, there are several losses that are created by operational risks. These can be characterized by seven event factors, which are recorded in the institution's financial statements consistent with Accounting and Auditing Organizations of Islamic Financial Institutions (AAOIFI):

- Internal fraud- Losses due to acts of a type intended to defraud misappropriated property or circumvent regulations, the laws or company policy, excluding diversity/discrimination events, *which involves at least an internal party.*
- External fraud- Losses due to acts of a type intended to defraud, misappropriate property or circumvent the laws, *by a third party.*
- Employment practices and workplace safety- Losses arising from acts inconsistent with employment, health or safety laws or agreements, from payment of personal injury claims, or from diversity / discrimination events.
- Clients, products, and business practices- Losses arising from an unintentional or negligent failure to meet a professional obligation to specific clients (including fiduciary and suitability requirements), or from the nature or design of a product.
- Damage to physical assets – Losses arising from loss or damage to physical assets from natural disaster or other events.
- Business disruption and system failures - Losses arising from disruption of business or system failures.

- Execution, delivery, and process management - Losses from failed transaction processing or process management, from relations with trade counterparties and vendors

In the recent years it has significant been proven that the operational risks are caused by events, which are mentioned above. Besides this, Islamic Islamic banks must also increase attention to social, shari'ah and environmental issues; issues that can result in operational risk loss. So, the scope of operational risk management has extended in monitoring and managing these risks as well.

Current Methods for Calculating Operational Risk

There are several ways for Islamic Islamic banks to measure operational risk within the framework that has been outlined by Islamic Financial Services Board. The following methods are described in the "International Convergence of Capital Measurement and Capital Standards, June 2004". In this document a framework that is outlined to present three methods for calculating operational risk capital charges in a continuum of increasing sophistication and risk sensitivity. The methods are as follows:

   i.   The Basic Indicator Approach (BIA)
   ii. The standardized Approach (TSA)
   iii. The Advanced Measurement Approach (AMA)

Islamic Islamic banks are also encouraged to move along the spectrum of available approaches, which are enumerated here in the order of the difficulty of adoption. The first two approaches are easier to adopt than the AMA approach. The AMA approach requires a large database of loss data. In contrast, the BIA and the TSA do not use operational loss data.

The Basic Indicator approach

The Basic Indicator Approach is the simplest, but it will charge the most capital generally. It is based on a straight percentage of gross income, which includes net income and net fee-income but excludes extraordinary or irregular items. While this approach may roughly capture the scale of an institution's operations, it surely has only the most questionable link to the risk of an expected loss due to internal or external events.

Islamic banks that uses the Basic Indicator Approach must hold capital for operational risk *equal to the average over the previous three years of a fixed percentage (denoted alpha) of positive annual gross income*. Figures for any year in which annual gross income is negative or zero, should be excluded from both the numerator and denominator when calculating the average. The charge may be expressed as follow:

$$K_{\text{BIA}} = \frac{\sum_{i=1}^{n}(GI_i * r)}{n} \tag{1}$$

Where:

K_{BIA} = The capital charged under the Basic Indicator Approach.

GI = Gross income, where positive, over the previous three years.

n = Number of the previous three years for which gross income is positive.

r = 15% (which is set by the committee, relating the industry wide level of required capital to the industry wide level of the indicator).

GI, the Gross income, will be defined as net income plus net fee-income, as is defined by national supervisors and/or national accounting standards. The intention is that this measure should:
- Be gross of any provisions (e.g. for unpaid intallments);
- Be gross of operating expenses, including fees paid to outsourcing service providers;
  (In contrast to fees paid for services that are outsourced, fees received by Islamic banks that provide outsourcing services shall be included in the definition of gross income);
- Exclude realized profits/losses from the sale of securities in the banking book;
  (Realized profits/losses from securities classified as "held to maturity" and "available for sale", which typically constitute items of the banking book (e.g. under certain accounting standards), are also excluded from the definition of gross income);
- Exclude extraordinary or irregular items as well as income derived from takaful.

For the Basic Indicator Approach, there are no criteria specified which Islamic banks has to satisfy. However Islamic banks that use this approach are encouraged to comply with the Committee's guidance on "Sound Practices for the Management and Supervision of Operational Risk, February 2003".

The Standardized Approach

The concept for applying the Standardized Approach is basically the same as the Basic Indicator Approach. The main difference between the two is that "The Standardized Approach" must divide the bank's business operations into 8 business lines (BLs): corporate finance, trading & sales, retail banking, commercial banking, payment & settlement, agency services, asset management, and retail brokerage.

Within each business line, gross income is a broad indicator that serves as an approximated scale for the business operations and thus the likely scale of operational risk exposure within each of these business lines. The capital charge for each business line is calculated by multiplying gross income by a factor (denoted beta) assigned to that

business line. Beta serves as a proxy for the industry-wide relationship between the operational risk loss experience for a given business line and the aggregate level of gross income for that business line. The Beta factors are displayed in table 5.

Table 1: Percentage of the relative weighting of the business lines

Business Lines	Beta Factors
Corporate finance (β_1)	18%
Trading and sales (β_2)	18%
Retail Banking (β_3)	12%
Commercial Banking (β_4)	15%
Payment and Settlement (β_5)	18%
Agency Services (β_6)	15%
Asset Management (β_7)	12%
Retail Brokerage (β_8)	12%

Within each business line, gross income is a broad indicator that serves as a proxy for the scale of business operations and thus the likely scale of operational risk exposure within each of these business lines. The capital charge for each business line is calculated by multiplying gross income by a beta factor assigned to that business line. Beta serves as a proxy for the industry-wide relationship between the operational risk loss experience for a given business line and the aggregate level of gross income for that business line. It should be noted that in the Standardised Approach gross income is measured for each business line, not the whole institution, i.e. in Corporate Finance, the indicator is the gross income generated in the Corporate Finance business line.

The total capital charge is calculated as the simple sum of the regulatory capital charges across each one of the business lines. The total capital charge may be expressed as:

$$K_{\text{TSA}} = \frac{\sum_{j}^{3} \max\left[\left(\sum_{i}^{8}\left(GI_i * \beta_i\right)\right),\ 0\right]}{3} \qquad (2)$$

K_{TSA} = The capital charge under the Standardized Approach.

GI_i = Annual gross income in a given year, as defined above in the Basic Indicator Approach, for each of the eight business lines.

β_i = A fixed percentage *set by the Committee*, relating the level of required capital to the level of the gross income for each of the eight business lines.

For using the Standardized Approach, there are certain criteria specified which Islamic banks has to satisfy, these are defined in the document: "International Convergence of Capital Measurement and Capital Standards, June 2004", in paragraph 660- 663.

The Advanced Measurement Approach

As one can see, the gross income is the basis for calculating a capital charge for both the Basic Indicator and Standardized Approaches. In practice, these two approaches calculate the most capital charges, compared to the Advanced Measurement Approach.

The Advanced Measurement Approach (AMA) is the last approach. This approach charges the least amount of capital; also this approach is comparatively more sophisticated. However, going by the sophistication of the AMA from the perspective of the cost beneficial factor, it will perhaps be wrong to conclude that it is thus far the best approach, for some Islamic banks. Consider that only large Islamic banks have the financial power to implement this approach and also make it profitable. The AMA, however, offers the greatest possibility to reduce capital requirements. It includes three approaches, namely the internal measurement approach (IMA), the scorecard approach and the Loss Distribution Approach.

Furthermore, on the application of these types of approaches several types of trade activities and several types of events are distinguished. These trade activities, which are referred as the 8 business lines, can be subdivided into sections. Within these sections several activities are grouped together. The mapping process is one the requirements which is set out by the Basle Committee.

The difference between the AMA method and other methods is that many data are collected. Also, Islamic banks develop several methods to analyze these data to determine a reasonable amount for the regulatory capital. There are currently three sub methods available for the Advanced Measurement Approach, i.e., Scorecard Approach, Internal Measurement Approach and Loss Distribution Approach.

Scorecard Approach

In the scorecard approach, Islamic banks initially determine a level of operational risk capital at the firm's business line and over time these amounts will be modified according to the Scorecard. Islamic banks aims to improve the risk control environment that will reduce both the frequency and severity of future operational risk losses. By identifying a number of risk indicators for particular risk types within business lines, one can captures the underlying risk profile of the various business lines. These risk indicators represents indirectly the altitude of the operational risk. A combination of risk indicator will be combined into a score, to allocate the altitude of the operational risk. After a certain time, the performance of these indicators will be assessed. Based on these assessments one can decide which point must still be improved. Also, based on the scorecard, one can analyze what was effectively the indirect influence of the indicators on eventual operational risk losses.

Where the Scorecard approach differs from other approaches (Internal Measurement Approach and Loss Distribution Approach) is that it relies less exclusively on historical loss data in determining capital amounts. Instead of this, after the size of the regulatory capital is determined, its overall size and its allocation across business lines will be modified on a qualitative basis. However, historical operational risk loss data must be used to validate the results of scorecards.

Internal Measurement Approach

The Internal Measurement Approach provides discretion to individual Islamic banks in the use of internal loss data. In this approach Islamic banks estimate the operational risk capital based on the measurement of the total expected losses. The IMA approach assumes a fixed, direct relationship between expected loss (the mean of the loss distribution) and the unexpected loss (the tail of the distribution). The relationship can be linear; this implies that the capital charge is a simple multiplication of the expected loss with a fixed number. Or non-linear, implying that total capital charge will be a more complex function of expected losses.

The IMA approach calculates the capital charge based on a framework that divides an Islamic bank's operational risk exposure into a series of business lines and events. In such a framework separate expected losses are calculated for each business line and event type combination. Such an approach, calculates the expected losses generally by estimating the loss frequency and the size of the amount for various business line and event combination by using internal loss data and, where appropriate, relevant external loss data, along with a measure of the scale of business activities for the particular business line in question.

While these elements can be specified in a variety of ways, in general they can be described as follow:
- PE: The probability that an operational risk event occurs over some future horizon.
- LGE: The average loss given that an event occurs.
- EI: An exposure indicator that is intended to capture the scale of the bank's activities in a particular business line.

The EI exposure indicator is specified by the supervisor for each type of business line an event combination. EI is a proxy for the size or amount of risk of each business line's operational risk.

The Expected loss (EL) for each business line and event combination will be calculated with the following formula:

$$EL = EI \ * PE \ * LGE \tag{3}$$

Combining these parameters, the IMA capital charge for each business line i and event type j combination K_{ij} would be:

$$K_{ij} = x_{ij} \ * \ EI_{ij} \ * PE_{ij} \ * \ LGE_{ij} = x_{ij} * \ EL_{ij} \qquad (4)$$

In this formula we expect a linear relationship between expected losses and the tail of the distribution. The parameter x_{ij} translates the estimates of expected losses, EL for the business line and event type combination into a capital charge. The x_{ij} for each business line and event type combination would be specified by the supervisor.

Loss Distribution Approach

Under the loss distribution approach, the Islamic banks estimate for each business line/event type, the probability distribution functions of the single event impact and the event frequency, for the next one year using its internal data. And it computes the probability distribution function of the cumulative operational risk loss. The capital charge is based on the sum of all operational risk for each business line/event type.

However, during the application of this approach correlation effect are not considered in this method. The advantage is this approach is that it can possibly increase the risk sensitivity. This method differs from the Internal Measurement Approach (IMA) in two important aspects. It aims to assess expected loss and unexpected loss directly and *without* making an assumption about the relationship between the expected loss and the unexpected loss. So, there is no need for the supervisor to determine a multiplication factor (gamma) under this approach. Also the bank itself determines the structure of business lines and event types.

The three methods above are summarized in Table 2.

Table 2: **Overview of Operational Risk Measures According to the Basel Committee on Banking Supervision (2003a, 2004a, 2005b and 2006b).**

Operational Risk Measure	Data Requirements	Regulatory capital charge	Remarks
Basic Indicator Approach (BIA)	A fixed percentage of average annual gross income over the previous three years.	[years(1-n) ($GIn*$)]/N, where GI = annual (positive) gross income, over the previous three years ("exposure factor"),	Figures for any year, in which annual gross income is negative or zero should be excluded from both the numerator and denominat

		n = number of the previous three years (N) for which gross income is positive, and = 15%, which is set	
Traditional) Standardized Approach (TSA)	The three-year average of the summation of the regulatory capital charges across each of the *business lines* (BLs) in each year.	{ years(1-3)*max[(*GI*1-8* 1-8),0]}/3, where GI1-8 = annual gross income for each of the eight BLs And 1-8 = fixed percentage relating the level of required capital to the level of the gross income for each of the eight BLs defined by the Basel Committee.	equals 18% for the BLs corporate finance, trading and sales, and payment and settlement; 15% for commercial banking, agency services; and 12% for retail banking, asset management, and retail brokerage.
Advanced Measurement Approaches (AMA)	Generated by the bank's internal operational risk measurement system.	AMA includes *quantitative* and *qualitative* criteria for the self-assessment of operational risk, which must be satisfied to ensure adequate risk management and oversight. The *qualitative* criteria center on the administration and regular review of a sound internal operational risk measurement system. The *quantitative* aspects of AMA include the use of internal data, (ii) external data, (iii) scenario analysis, and (iv) business environment and internal control factors subject to the *AMA soundness standard* and requirements for *risk mitigation* and *capital adjustment*	Under the *AMA soundness standard*, a bank must be able to demonstrate that its operational risk measure is comparable to that of the internal ratings-based approach for credit risk, i.e., a one-year holding period and a 99.9th percentile confidence interval. Islamic banks are also allowed to adjust their total operational risk up to 20% of the total operational risk capital charge.

Extreme Value Theory: A Suggested Approach for Measuring Operational Risk

The extreme value theory (EVT) offers methods for modelling "fat tails" or "heavy tails" of a distribution ("let the tails speak for themselves"). In the context of operational risks, interest focuses on stress losses about whose distribution the VaR approach does not provide any information. The VaR of market risk is usually based on the assumption of a normal distribution. This assumption considerably facilitates calculations and in most cases, provides a good approximation to the actual distribution. In the case of operational risks, however, the distribution has a right skew. Within the EVT, the generalized extreme value distribution (GEV) and the generalized Pareto distribution are more suitable statistical instruments. "Classic" EVT describes the distribution of maxima and minima of a sample. The GEV represents the distribution of normalized maxima. After appropriate normalization, there are three possibilities for their asymptotic distribution: the Gumbel, Frechet and Weibull extreme value distributions.

In the literature, EVT is often used to estimate very high quantiles, for instance to compute Value-at-Risk figures (see McNeil (2000), or Këllezi and Gilli, (2003)). But estimating an extreme quantile of a distribution is very different from obtaining the whole Pareto Distribution Funtion (PDF) of the losses, which is nevertheless needed in order to compute the convolution of the severity distribution with itself (this is how we get the aggregated loss distribution). In addition, the global shape of this distribution is also important when dealing with dependence measurement techniques.

Concerning the tail area, quite a number of different distributions could be adopted; for example, LogNormal and Pareto curves are commonly accepted in insurance to model large claims. However, in this analysis, extreme distributions, stemming from the Extreme Value Theory (EVT), are utilized. The reason lies in the fact that EVT has solid foundations in the mathematical theory of the behavior of extremes and, moreover, many applications have indicated that EVT appears to be a satisfactory scientific approach in treating rare, large losses. It has been widely applied in structural engineering, oceanography, hydrology, reliability, total quality control, pollution studies, meteorology, material strength, highway traffic and, more recently, in the financial and insurance fields[5]. For a comprehensive source on the application of EVT to finance and insurance, see Embrechts *et al.* (1997), and Reiss and Thomas (2001).

In general, operational risk losses undoubtedly present characteristics analogous to data originating from the above-mentioned fields (immediate analogies, for example,

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[5] In recent years, there have been a number of extreme value studies and applications in finance and insurance: for example McNeil studies the estimation of the tails of loss severity distributions (1997), examines the quantile risk measures for financial time series (1998) and provides an extensive overview of the extreme value theory for risk managers (1999); Embrechts studies the potentials and limitations of the extreme value theory (1999 and 2000); McNeil and Frey study the estimation of tail-related risk measures for heteroschedastic financial time series (2000).

can be found in insurance, reinsurance, reliability and total quality control). In fact, operational risk data appear to be characterized by two "souls": the first one, driven by high-frequency low-impact events, constitutes the body of the distribution and refers to expected losses; the second one, driven by low-frequency high-impact events, constitutes the tail of the distribution and refers to unexpected losses. In practice, the body and the tail of data do not necessarily belong to the same, underlying, distribution or even to distributions belonging to the same family. More often their behavior is so different that it is hard to identify a unique traditional model that can at the same time describe, in an accurate way, the two "souls" of data: the conventional inference on the BLs whole data sets furnishes a clear proof of that[6].

Consequently, in all the cases in which the tail tends "to speaks for itself", EVT appears to be an useful inferential instrument with which to investigate the large losses, owing to its double property of focusing the analysis only on the tail area (hence reducing the disturbance effect of the small/medium-sized data) and treating the large losses by an approach as scientific as the one driven by the Central Limit Theorem for the analysis of the high-frequency low-impact losses[7]. Clearly, EVT is not a "panacea", since specific conditions are required for its application and even then it is still open to some criticisms, extensively investigated in the literature (on this topic, see for example Embrechts *et al.* (1997), Diebold *et al.* (1998), and Embrechts *et al.* (2003)).

Unlike traditional methods, EVT does not require particular assumptions on the nature of the original underlying distribution of all the observations, which is generally unknown. EVT is applied to real data in two related ways. The first approach (see Reiss and Thomas, 2001, p. 14 ff) deals with the maximum (or minimum) values the variable takes in successive periods, for example months or years. These observations constitute the extreme events, also called block (or per-period) maxima. At the heart of this approach is the "three-types theorem" (Fisher and Tippet, (1928)), which states that there are only three types of distributions which can arise as limiting distributions of extreme values in random samples: the Weibull type, the Gumbel type and the Frechet type. This result is very important, since the asymptotic distribution of the maxima always belongs to one of these three distributions, regardless of the original one. Therefore the majority of the distributions used in finance and actuarial sciences can be divided into these three classes, according to their tail-heaviness:

•   light-tail distributions with finite moments and tails, converging to the

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[6] Some mixture distributions could be investigated in order to identify a model that provides a reasonable fit to both the body and the tail of data. However, the disadvantage of such distributions is that they are more complex and, hence, less easy to handle. Furthermore a mixture model would be an arbitrary choice, not supported by a robust theory and, because of that, one would have less confidence in extrapolating the outcomes beyond the empirical data.

[7] To cite, respectively, Diebold *et al.* (*1998*), and Smith (1987),, *"EVT helps the analyst to draw smooth curves through the extreme tails of empirical survival functions in a way that is guided by powerful theory and hence provides a rigorous complement to alternatives such as graphical analysis or empirical survival functions"* and *"There is always going to be an element of doubt, as one is extrapolating into areas one doesn't know about. But what EVT is doing is making the best use of whatever data you have about extreme phenomenon".*

Weibull curve (Beta, Weibull);

- medium-tail distributions for which all moments are finite and whose cumulative distribution functions decline exponentially in the tails, like the Gumbel curve (Normal, Gamma, LogNormal);

- heavy-tail distributions, whose cumulative distribution functions decline with a power in the tails, like the Frechet curve (T-Student, Pareto, LogGamma, Cauchy).

The Weibull, Gumbel and Frechet distributions can be represented in a single three parameter model, known as the Generalised Extreme Value distribution (GEV):

$$GEV_{\xi\mu\tau(x)} = \exp\left\{-\left(1+\xi\frac{x-\mu}{\tau}\right)^{1/\xi}\right\} \quad if \ \xi \neq 0$$

$$\exp\left\{-\exp\left\{\frac{x-\mu}{\tau}\right\}\right\} \quad if \ \xi \neq 0$$

(5)

Where $1+\xi x > 0$

The parameters μ and $\quad$ correspond to location and scale; the third parameter, $\quad$, called the shape index, indicates the thickness of the tail of the distribution. The larger the shape index, the thicker the tail. The second approach to EVT (see Reiss and Thomas, 2001, p. 23 ff) is the Peaks Over Threshold (POT) method, tailored for the analysis of data bigger than preset high thresholds.

The severity component of the POT method is based on a distribution (Generalised Pareto Distribution - GPD), whose cumulative function is usually expressed as the following two parameter distribution:

$$GPD_{\xi\tau(x)} = 1-\left(1+\xi\frac{x}{\tau}\right)^{-\frac{1}{\xi}} if \ \xi \neq 0$$

$$1-\exp\left\{-\frac{x}{\tau}\right\} if \ \xi \neq 0$$

(6)

Where $x \geq 0 \ if \ \xi \geq 0, \ 0 \leq x \leq -\tau/\xi \ \xi < 0$ and and $\quad$ and $\quad$ represent respectively the shape and the scale parameter. It is possible to extend the family of the GPD distributions by adding a location parameter μ. In this case the GPD is defined as:

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$$GPD_{\langle \sim \dagger (x)} = 1 - \left(1 + \langle \frac{x - \sim}{\dagger}\right)^{-\frac{1}{\langle}} \quad if \quad \langle \neq 0$$

$$1 - \exp\left\{-\frac{x - \sim}{\dagger}\right\} if \quad \langle \neq 0$$

(7)

The interpretation of   in the GPD is the same as in the GEV, since all the relevant information on the tail of the original (unknown) overall distribution is embedded in this parameter[8]: when   < 0 the GPD is known as the Pareto "Type II" distribution, when   =0 the GPD corresponds to the Exponential distribution. The case when   > 0 is probably the most important for operational risk data, because the GPD takes the form of the ordinary Pareto distribution with tail index   = 1/   and indicates the presence of heavy-tail data[9]; in this particular case there is a direct relationship between and the finiteness of the moments of the distribution:

$$E\left(x^{k}\right) = \infty \quad if \quad k \geq 1/\langle$$

For instance, if   0.5 the GPD has an infinite variance, if   1 there is no finite moment, not even the mean. This property has a direct consequence for data analysis: in fact the (heavier or lighter) behavior of data in the tail can be easily directly detected from the estimate of the shape parameter.

Now, let $F_x(x)$ be the (unknown) distribution function of a random variable X (with right-end point x_F) which describes the behaviour of the operational risk data in a certain BL and let $F_u(y)$ be its excess distribution at the threshold u. The excess distribution can be introduced as a conditional distribution function, that is:

$$F_u(y) = P\left(X - u \leq y \mid X > u\right) = \frac{F_x(x) - F_x(u)}{1 - F_x(u)}$$

(8)

for y=x-u>0

It represents the probability that a loss exceeds the threshold u by at most an amount y, given that it exceeds the threshold. The theory (Balkema-De Haan (1974), and Pickands (1975)) maintains that for a large class of underlying distributions, the excess distribution $F_u(y)$ converges asymptotically to a GPD as the threshold is progressively raised to the right endpoint x_F of the distribution.[10]

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[8] The maxima of samples of events from GPD are GEV distributed with shape parameter equal to the shape parameter of the parent GPD. There is a simple relationship between the standard GDP and GEV such that GPD(x) = 1+log GEV(x) if log GEV(x) > -1
[9] The ordinary Pareto is the distribution with distribution function F(x) = 1 - (a/x) and support x > a. This distribution can be rewritten as F(x) = 1 - (1 + (x - a)/a)   so that it can be seen to be a GPD with shape   = 1 / , scale   = $a/$ and location μ = a. In practice it is a GPD where the scale parameter is constrained to be the shape multiplied by the location, hence it is a little less flexible than a GPD, where the scale can be freely chosen.
[10] The conditions under which excess losses converge to GPD distributions are very large.

$$\lim_{u \to x_F} \sup \left| F_u(y) - GPD_{\xi,s}(y) \right| = 0 \tag{9}$$

Where

$$GPD_{\xi\uparrow(x)} = 1 - \left(1 + \xi \frac{y}{s}\right)^{-\frac{1}{\xi}} \quad if \quad \xi \neq 0 \tag{10}$$

$$1 - \exp\left\{-\frac{y}{s}\right\} if \quad \xi \neq 0$$

with: y= x-u = excess, = shape, = scale;
and support $y \in [0, x_F - u]$ if $\quad 0$

$$y \in [0, -S/\xi] \text{ if } \quad 0$$

In this work, the $GPD_{\xi,s}(y)$ will be called the "excess GPD", to stress the fact that the argument y represents the excesses, that is to say the exceedances x (i.e. the data larger than the threshold u) minus the threshold u itself.

Equivalently, the limit condition (6) holds if the exceedances x are used in place of the excesses y: changing the argument, the $F_u(y)$ and $GPD_{\xi,s}(y)$ transform respectively to $F_u(x)$ and $GPD_{\xi,\sim,s}(x)$, with the threshold u, now, representing the yendpoint x_F, the exceedance distribution $F_u(x)$ converges asymptotically to a GPD with the same shape £ scale and location $\mu = u$. The $GPD_{\xi,\sim,s}(x)$ will be called the "exceedance GPD" because it deals with the exceedances x at u.

One of the most important properties of the GPD is its stability under an increase of the threshold.To show that, let isolate $F_x(x)$ from (9):

$$F_x(x) = [1 - F_x(u)]F_u(y) + F_x(u) \tag{11}$$

Looking at the limit condition (11), both the excess distribution $F_u(y)$ and the exceedance distribution $F_u(x)$ can be approximated well by suitable GPDs. By using the "exceedance GPD", one obtains

$$F_x(x) = [1 - F_x(u)]GPD_{\xi,\sim,s}(x) + F_x(u) \tag{12}$$

Substituting the $GPD_{\xi,u,s}$ expression in (8):

$$F_x(x) = \left[1 - F_x(u)\right]\left[1 - \left(1 + \xi\,\frac{x-u}{s}\right)^{-\frac{1}{\xi}}\right] + F_x(u) \tag{13}$$

The only element now required to identify $F_x(x)$ completely is $F_x(u)$, that is to say the value of the (unknown) distribution function in correspondence with the threshold u. To this end, the empirical estimator of $F_x(x)$, computed at u, can be a viable solution:

$$F_n(u) = \frac{1}{n}\sum_{i=1}^{n} 1_{(x \leq u)} = \frac{n - n_u}{n} \tag{14}$$

where: n is the total number of observations

n_u the number of observations above the threshold u

The threshold u should be set at a level that let enough observations exceeding u to obtain a reliable empirical estimate of $F_x(u)$. Consequently, $F_x(x)$ can be completely expressed by the parameters of the and the number of observations (total and over the threshold):

$$F_x(x) = \frac{n_u}{n}\left[1 - \left(1 + \xi\,\frac{x-u}{s}\right)^{-\frac{1}{\xi}}\right] + \left(1 - \frac{n_u}{n}\right) \tag{15}$$

which simplifies to

$$F_x(x) = \frac{n_u}{n}\left[1 - \left(1 + \xi\,\frac{x-u}{s}\right)^{-\frac{1}{\xi}}\right] \tag{16}$$

This quantity is defined as the "tail estimator" of $F_x(x)$, as it is valid only for $x > u$. It is possible to demonstrate that the "tail estimator" is also GPD distributed: it is the semiparametric representation of the $GPD_{\xi,\sim,\dagger}$ referred to all the original data, with the same shape ξ and location and scale equal to μ and respectively. The $GPD_{\xi,\sim,\dagger}$ will be called the "full GPD" because it is fitted to all the data in the tail area. Semiparametric

estimates for the "full GPD" parameters can be derived from those of the "exceedance GPD":

$$\dagger = S\left(\frac{n_u}{n}\right)^{<} \tag{17}$$

$$\sim = u - \frac{S}{<}\left[1-\left(\frac{n_u}{n}\right)^{<}\right] \tag{18}$$

As there is a one-to-one relationship between the "full GPD" ($GPD_{<,u,\dagger}$) and the "exceedance GPD" ($GPD_{<,u,S}$), it is also possible to express the scale parameter of the latter by the former: $= + (u\text{-}\mu)$. It should be noted that, while the scale (S) of the "exceedance GPD" depends on where the threshold is located, the shape ($<$), the location (μ) and scale ( ) of the "full GPD" are independent of the threshold. Hence a nice practical method to check the robustness of the model for some specific data is to evaluate the degree of stability of these latter parameters over a variety of thresholds. By applying the GPD stability property, it is possible to move easily from the excess data (y = x-u) to the tail of the original data (x > u) and from the excess distribution $F_u(y)$ to the underlying (unknown) distribution $F_x(x)$.

An immediate consequence of the GPD stability is that if the exceedances of a threshold u follow a $GPD_{<,u,S}$, the exceedances over a higher threshold v > u are $GPD_{<,u,S+<(v-u)}$, that is they are also GPD distributed with the same shape £ the location equal to v (the new threshold) and the scale equal to $+$ $(v\text{-}u)$. This property will be extensively adopted in the current exercise.

Conclusions

Operational Risk is significant in Islamic banks. There is a large size 'reporting bias' in public datasets requiring special treatment before analysis to reduce overestimation of capital based on the current methods. For these databases, GPD is an "appropriate model" to represent tail severity for all business lines. Because, this method is able to capture the wide differences in 'riskiness' of different Business Lines and also the differences in 'event type' classification across databases. However, the insufficient data for conclusive analysis on CAS Loss Event Type. And also the supplementing 'internal' data with 'external' data can significantly improve operational risk models.

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