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ASYMMETRIC VOLATILITY AND RISK ANALYSIS OF BITCOIN CRYPTOCURRENCY MARKET

(Kemeruapan Asimetrik dan Analisis Risiko untuk Pasaran Mata Wang Kripto Bitcoin)

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ABSTRACT

This study provides an estimation of Bitcoin's volatility using a variation of GARCH (volatility) models. The Box-Jenkins Procedure is used throughout the analysis. The volatility clustering effect is found in Bitcoin, which suggests that GARCH models are applicable in its return series. In the empirical analysis, the standard errors of cryptocurrency returns are assumed to follow a Student-*t* distribution for the best fitting model. The Glosten, Jagannathan, and Runkle (GJR)-GARCH(1,1) model shows that Bitcoin's log return series exhibits an inverted leverage effect, where the volatility of Bitcoin's return tends to increase when good news happens. In financial applications, the accuracy of volatility estimation and forecasting is crucial in providing a reliable tool for risk management, option trading, asset pricing, among others. The value-at-risk measurement transforms the estimated GARCH volatility into the maximum potential loss at a certain level of confidence (95% or 99%). By including the COVID-19 period in our empirical study, we found that the pandemic has a positive impact on cryptocurrency markets. This finding provides useful information to investors in their risk management and portfolio analysis.

Keywords: cryptocurrencies return; GARCH; volatility models; value-at-risk

ABSTRAK

Kajian ini menyediakan anggaran pulangan Bitcoin menggunakan variasi model kemeruapan GARCH. Prosedur Box-Jenkins digunakan sepanjang analisis. Kesan pengkelompokan kebolehubah ditemui dalam Bitcoin, yang menunjukkan bahawa model GARCH adalah sesuai dalam model siri pulangan. Dalam analisis empirik, ralat piawai pulangan mata wang kripto dianggap mengikut taburan Student-t bagi model yang paling sesuai. Model Glosten, Jagannathan, dan Runkle (GJR)-GARCH(1,1) menunjukkan bahawa siri pulangan log Bitcoin menunjukkan kesan pembebanan terbalik, di mana kemeruapan pulangan Bitcoin cenderung meningkat apabila berita baik pasaran diumumkan. Dalam aplikasi kewangan, ketepatan anggaran kebolehubah dan ramalan adalah penting dalam menyediakan alat yang boleh dipercayai untuk pengurusan risiko, perdagangan opsyen, penetapan harga aset, dan lain-lain. Pengukuran Nilai Risiko mengubah anggaran kemeruapan GARCH yang dianggarkan menjadi kerugian maksimum potensi pada tahap keyakinan tertentu (95% atau 99%). Dengan menglingkupi tempoh COVID-19 dalam kajian empirik, adalah dapati bahawa pandemik mempunyai impak positif terhadap pasaran mata wang kripto. Penemuan ini menyediakan maklumat berguna kepada pelabur dalam pengurusan risiko dan analisis portfolio.

Kata kunci: pulangan mata wang kripto; GARCH; model kemeruapan; nilai risiko

1. Introduction

In recent years, the popularity and use of cryptocurrencies has increased dramatically. The surge of interest in cryptocurrencies brings extensive literature on this topic. However, it is well known that cryptocurrencies are highly volatile (Bakas *et al.* 2022; Aharon *et al.* 2023; Panagiotidis *et al.* 2022) compared to traditional markets. In order to capture the volatility behavior of cryptocurrency markets, numerous statistical methods including machine learning approaches (Amirshahi & Lahmiri 2023; D'Amato *et al.* 2022; Oyedele *et al.* 2023) have been introduced over the years. A good model is highly desired to accurately forecast the volatility and provide precise information to determine market risk. Therefore, understanding the volatility of cryptocurrencies is crucial in portfolio management and risk applications. The pioneer of decentralized cryptocurrency, Bitcoin was designed in 2008 based on blockchain technology. Bitcoin was created to facilitate electronic payments between individuals without going through a third party. It can be freely traded on digital exchanges and have no central bank or another financial institution standing behind them. This feature lets cryptocurrencies to attract lots of attention. In 16 November 2021, the global crypto market capitalization is $2.63 Trillion with a 9.79% decrease over the last day. The total crypto market volume over the last 24 hours is $147.44 Billion. The two dominant cryptocurrencies in the market are Bitcoin and Ethereum. In 13 August 2021, Bitcoin and Ethereum dominance stand at 43.2% and 18.8% respectively. The price analysis of Bitcoin and Ethereum (CoinMarketCap 2021) are representative as the trend of cryptocurrencies market. In 12 March 2020, the price of Bitcoin drops from $7913.62 to $4970.79 within one day, decrease by 37.19%. This shows that the price of Bitcoin can fluctuate dramatically especially during the COVID-19 period (Apergis 2022). The impact of COVID-19 on cryptocurrency markets has also been observed in studies by Foroutan and Lahmiri (2022) and Balcilar *et al.* (2022) for ten and seven major cryptocurrencies. In this study, we re-examine the possible impact of COVID-19 on Bitcoin. Besides COVID-19, under the condition of thin trading volume (Komarraju 2021) and an imbalance of buyers compared to sellers or vice versa, the cryptocurrency markets may encounter a crash. In addition, after influential person Elon Musk's decision (Beatrice 2021) not to accept Bitcoin for Tesla, Bitcoin's price immediately plunged by 12%. Thus, the cryptocurrency market is still considered a high-volatility or high-risk investment. Therefore, a study on its volatility is crucial in determining the risk.

Volatility is an important factor in options trading. Here volatility refers to the conditional variance of the underlying asset return. However, the volatility is not directly observable by one data. Normally, volatility represents how large an asset's return swing around the predicted return. The empirical findings of financial return time series give some common characteristics of volatility (Bakas *et al.* 2022). First, there exist volatility clusters, that is, volatility may be high for a certain time periods and low for other periods. Second, volatility evolves over time in a continuous manner - that is, volatility jumps are rare. Third, volatility seems to react differently to a big price increase or a big price drop. By the leverage effect, the volatility should be higher when the price drops due to the higher debt-equity ratio (Tsay 2002). The immediate application of volatility analysis is determining market risk. The common methods for quantifying cryptocurrency market risks are value-at-risk (Liu *et al.* 2020; Silahli *et al.* 2021) and expected shortfall (Acereda *et al.* 2020; Malek *et al.* 2023).

The objective of this study is to examine asset return models for Bitcoin, focusing on the volatility models. These volatility models are referred to as various conditional heteroscedastic models, which means the conditional variance of return series change over the time. In addition, the impact of COVID-19 to the cryptocurrency market is also investigated using a dummy variable in the volatility models. Finally, the findings in the volatility analysis will be used to determine the multiple steps ahead forecasts of value-at-risk.

2. Methodology

Daily closing price for Bitcoin and Ethereum are collected from 1 Jan 2016 to 31 Dec 2021, which corresponds to 2192 observations. The data are available at Yahoo Finance website. The general volatility model of return series can be written as

$$r_t = \mu_t + \varepsilon_t \qquad \varepsilon_t = \sigma_t z_t \tag{1}$$

where μ_t is the mean that can be estimated by ARMA model and z_t are assumed to be identical and independently distributed with mean 0 and variance 1. In practice, z_t is often assumed to follow standard normal or standardized Student-*t* distribution. There are several ways (models) to predict σ_t.

The Glosten, Jagannathan, and Runkle (GJR)-GARCH (1993) is a asymmetric volatility model responds differently to positive and negative effect. It is well known that the price of a financial asset responds differently to positive and negative shocks. The bull walks up the stairs and the bear jumps out the window. However, a GARCH model fails to detect this kind of characteristics. In short, the GJR model is commonly used to detect the impact of news (whether good or bad) on a certain financial market. The GJR-GARCH(1,1) model assumes

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma I_{t-1} \varepsilon_{t-1}^2 \tag{2}$$

where $I_{t-1} = 1$ if $r_{t-1} < 0$ and $I_{t-1} = 0$ otherwise. γ represents the asymmetric effect. Normally, a significant $\gamma > 0$ means bad news ($r_{t-1} < 0$) tend to bring larger shocks than good news. Although the World Health Organization announced the first disease outbreak news on this new virus on 5 Jan 2020, we have selected the period that includes the COVID-19 period starting from 1 Feb 2020, which is after the virus started to spread globally. In order to study the COVID-19 effect on the volatility, we define an indicator variable

$$I_t = \begin{cases} 1, & \text{if } t \text{ is during COVID-19 period} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

and modify the GARCH(1,1) model to

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + I_t \left(\tilde{\alpha}_0 + \tilde{\alpha}_1 \varepsilon_{t-1}^2 + \tilde{\beta}_1 \sigma_{t-1}^2 \right) \tag{4}$$

Value-at-Risk (VaR) is the maximal loss of a financial position during a given time period for a given probability. Consider the same AR(p)-GARCH(1,1) model, if the z_t are assumed to follow standard normal distribution, the conditional distribution of r_{h+1} given all information available at forecast origin h is $N[\hat{r}_h(1), \hat{\sigma}_h^2(1)]$. Therefore, the one step ahead VaR with 5% quantile of distribution is

$$\hat{r}_h(1) + z_{0.05} \hat{\sigma}_h(1) = \hat{r}_h(1) - 1.65 \hat{\sigma}_h(1) \tag{5}$$

However, since most financial data are heavy-tailed, it is often assumed that z_t follows a standardized Student-*t* distribution with v degrees of freedom, then the 5% quantile used to calculate the 1-period horizon VaR at time index h is

$$\hat{r}_h(1) + \frac{t_{0.05,\upsilon} \hat{\sigma}_h(1)}{\sqrt{\upsilon / (\upsilon - 2)}} \tag{6}$$

where $t_{0.05,v}$ is the 5% quantile of a Student-t distribution with v degrees of freedom. The result obtained by Eq. (6) shows that we have 95% of chance not losing beyond that return. The variable of interest is the k-period log return at the forecast origin h, denoted by $r_h[k]$,

$$r_h\left[k\right] = \hat{r}_h(1) + \hat{r}_h(2) + ... + \hat{r}_h(k) \tag{7}$$

We are also interested in the associated forecast error for k-period log return forecast, denoted by $e_h[k]$,

$$e_h\left[k\right] = e_h(1) + e_h(2) + ... + e_h(k) \tag{8}$$

Following the AR(p)-GARCH(1,1) model, with the mean, $\hat{r}_h(l)$ and the variance of forecast error, $Var[e_h(l)]$, the conditional mean forecast of k-period log return and the variance of forecast error for $r_h[k]$ is

$$E\left(r_h\left[k\right] \mid F_h\right) = \sum_{l=1}^{k} \hat{r}_h(l) \tag{9}$$

$$Var(e_h[k]|F_h) = Var\left(\sum_{l=1}^{k} e_h(l)\right) = \sum_{l=1}^{k} Var(e_h(l)) \tag{10}$$

3. Result

2192 daily closing price of Bitcoin (USD) from 1 Jan 2016 to 30 Dec 2021 were collected in Yahoo Finance. We use the daily log return of Bitcoin as our main metric to study. Most studies are focused on short-term forecasting and risk evaluation. For long-term forecasting, one may use a weekly or monthly dataset. The estimated GARCH(1,1) model gives

$$r_t = 0.195 + \varepsilon_t$$
$$\sigma_t^2 = 0.084 + 0.903\sigma_{t-1}^2 + 0.097\varepsilon_{t-1}^2$$

where z_t follows t-distribution with estimated 3.37 degrees of freedom. The standard error of the parameter in the mean equation is 0.05, whereas those of the parameters in volatility equation are 0.03, 0.01 and 0.01 respectively. To investigate the COVID-19 effects on the volatility model, the duration of Bitcoin price data are limited to four years pre-COVID-19 period, that is Jan 2016 to Jan 2020. The estimated GARCH(1,1) model in this duration gives

$$r_t = 0.179 + \varepsilon_t$$
$$\sigma_t^2 = 0.077 + 0.888\sigma_{t-1}^2 + 0.112\varepsilon_{t-1}^2$$

where z_t has a t-distribution with estimated degrees of freedom 3.28. Apparently, the volatility equations do not have much difference with the previous estimation results. However, it is found that there is a positive shift in volatility model from 0.077 to 0.084 which indicated that the COVID-19 has a positive influence to the volatility of Bitcoin in general. Since there were no significant global events during this period, we assume that the positive impact on long-term volatility is caused by COVID-19. Therefore, the volatility model that included COVID-19 period is not influential and applicable to be used. For asymmetric analysis, the estimated GJR-GARCH(1,1) model is

$$r_t = 0.197 + \varepsilon_t$$
$$\sigma_t^2 = 0.058 + 0.108\varepsilon_{t-1}^2 + 914\sigma_{t-1}^2 - 0.045 I_{t-1}\varepsilon_{t-1}^2$$

where $I_{t-1} = 1$ if $r_{t-1} < 0$ and $I_{t-1} = 0$ otherwise. The asymmetric (leverage) effect, $\hat{\gamma} = 0.045$ is significant at p-value equals to 0.008, with standard error 0.017. The standard error of other parameters are 0.05, 0.03, 0.001 and 0.02 respectively, which are all significant at the 5% level. The AIC value of GJR-GARCH(1,1) model is 11290, which is slightly higher compared with GARCH(1,1) model. Therefore, we have considered that the GJR-GARCH model is statistically better than the GARCH model. However, we have used both models in the forecast evaluation for comparison.

For market risk analysis, let h be the forecast origin which represents the day 30 Dec 2021. The 5% quantile of the conditional distribution of r_{h+1} is

$$\hat{r}_h(1) + \frac{t_{0.05,\upsilon}\hat{\sigma}_h(1)}{\sqrt{\upsilon/(\upsilon-2)}} = 0.1968 + \frac{(-2.2459)\sqrt{8.6896}}{\sqrt{3.3946/(3.3946-2)}} = -4.0467$$

In 1 Jan 2022, there are 95% probability that investors will not lose more than 4.05% in Bitcoin return. The overall quantile estimation of in-sample forecast return is illustrated in Figure 1.

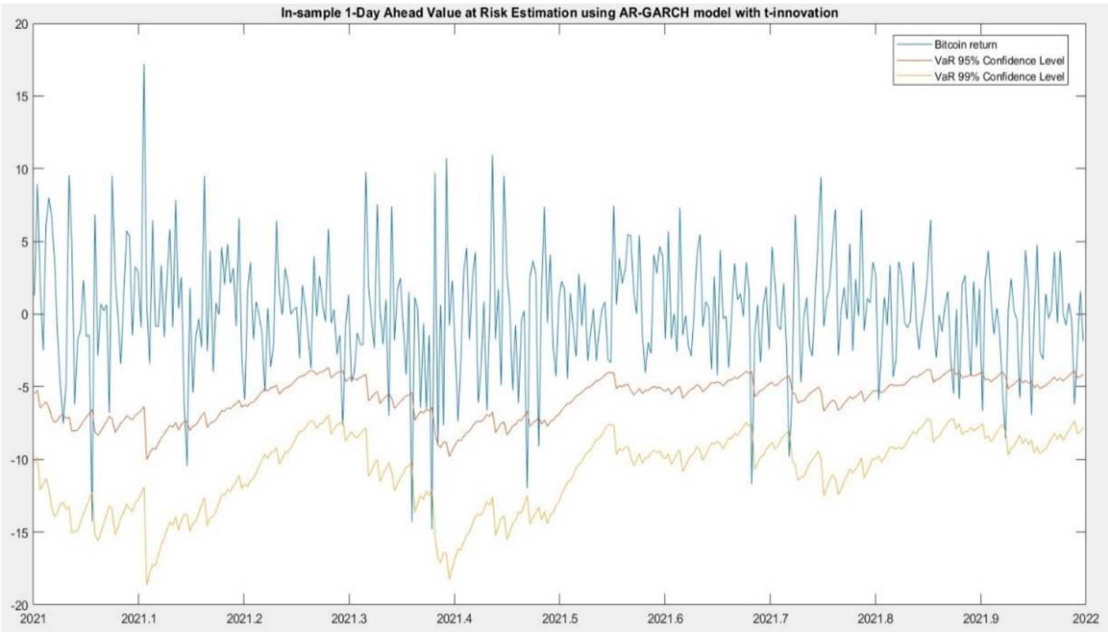

Figure 1: 5% and 1% quantile estimation of in-sample forecast return

If \$1 million are invested in Bitcoin, the corresponding 5 %VaR for 1-day horizon is

$$VaR = \$1,000,000 \times 0.040467 = \$4,0467$$

Using the similar calculation, the 1% quantile of the conditional distribution of r_{h+1} is –7.6339. The 1% VaR for 1-day horizon is \$76,339. For multiple days ahead forecast, the 5% quantile of the conditional distribution of $r_h[k] = r_{h+1} + ... + r_{h+k}$ is

$$\hat{r}_h[k] + \frac{t_{0.05,\upsilon} \times \sqrt{Var(e_h[k] \mid F_h)}}{\sqrt{\upsilon/(\upsilon-2)}} \tag{11}$$

where $e_h[k] = e_{h+1} + ... + e_{h+k}$ is the k-period forecast error. Assume the invested amount is \$1,000,000 in Bitcoin for long position, the VaR details are shown below:

Table 1: 5% and 1% VaR summary for k = 1, 7, 14, 30 days ahead

k days ahead	5% quantile	5% VaR	1% quantile	1% VaR
1	–4.0467	\$40,467	–7.6339	\$76,339
7	–9.9602	\$99,602	–19.5450	\$195,450
14	–13.4597	\$134,597	–27.1678	\$271,678
30	–18.4268	\$184,268	–38.9964	\$389,964

From Table 1, the result shows that the potential loss of holding Bitcoin for 30 days is \$184,268 or less, given probability 95%.

4. Discussion and Conclusion

There are several interesting findings in this cryptocurrency market analysis. Firstly, this study suggests that the GARCH(1,1) model with a Student-*t* distribution in standard error is the best elementary volatility model among all GARCH(q, p) models. The study also observes volatility clustering and heavy-tailed characteristics in the Bitcoin series. This implies that the volatility is not independent, although the noise is somewhat independent. The heavy-tailed property indicates that the return series of Bitcoin is better fitted in a fat-tailed distribution with relatively more extreme values compared to a normal distribution. This may avoid the issue of underestimation by the normality assumption in market risk determination.

Secondly, the study captures the asymmetric news impact by the GJR-GARCH(1,1) model. The Bitcoin return series exhibits an inverted leverage effect, meaning that the volatility of Bitcoin return tends to increase when good news happens. This is different from most of the stock markets or commodity markets where leverage effect (bad news impact) is commonly observed globally.

Thirdly, the study re-examines the impact of COVID-19 in the Bitcoin market and finds a positive impact on Bitcoin's volatility, although it is not very significant in terms of magnitude. It seems that COVID-19 has less impact on the cryptocurrency of Bitcoin.

Finally, the study computes the value-at-risk for various day-ahead, and these findings provide useful information for investors in their portfolio investments and risk management.

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