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COMMON FIXED POINT RESULTS FOR BOYD-WONG AND MEIR-KEELER CONTRACTION IN $\mathcal{F}$-METRIC SPACES

(*Hasil Titik Tetap Sepunya untuk Pemetaan Pengecutan Boyd-Wong dan Meir-Keeler dalam Ruang $\mathcal{F}$-Metrik*)

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ABSTRACT

A new notion of metric space generalization has been defined by Jleli and Samet, namely $\mathcal{F}$-metric space, in 2018. The objective of this study is to prove the existence and uniqueness of a common fixed point in the context of $\mathcal{F}$-metric space. We construct theorems of a common fixed point for commuting mapping pairs with Boyd-Wong and Meir-Keeler contraction in this space. Moreover, we extend the results of Park and Bae (1981) and Bera et al. (2022) to common fixed point theorems and $\mathcal{F}$-metric space, respectively. The Boyd-Wong contraction is attractive to discuss since we cannot apply the metrizability result on the $\mathcal{F}$-metric space to prove the theorem. The Meir-Keeler contraction is also interesting since it is a generalization of the Boyd-Wong contraction. Lastly, we provide an example of each case to support the findings of our study.

Keywords: $\mathcal{F}$-metric space; commuting mappings; common fixed point; Boyd-Wong contraction; Meir-Keeler contraction

ABSTRAK

Tanggapan baru bagi pengitlakan ruang metrik telah ditakrifkan oleh Jleli dan Samet, iaitu ruang $\mathcal{F}$-metrik, pada tahun 2018. Objektif kajian ini adalah untuk membuktikan kewujudan dan keunikan titik tetap sepunya dalam konteks ruang $\mathcal{F}$-metrik. Kami membina teorem titik tetap sepunya untuk pasangan pemetaan kalis tukar tertib dengan pemetaan pengecutan Boyd-Wong dan Meir-Keeler dalam ruang ini. Selain itu, kami melanjutkan keputusan Park dan Bae (1981) dan Bera et al. (2022) kepada teorem titik tetap sepunya dan ruang $\mathcal{F}$-metrik, masing-masing. Pemetaan pengecutan Boyd-Wong menarik untuk dibincangkan kerana kita tidak boleh menggunakan hasil kebolehmetrikan pada ruang $\mathcal{F}$-metrik untuk membuktikan teorem berkenaan. Pemetaan pengecutan Meir-Keeler juga menarik kerana ia merupakan pengitlakan bagi pemetaan pengecutan Boyd-Wong. Akhir sekali, kami menyediakan contoh bagi setiap kes untuk menyokong dapatan kajian.

Kata kunci: ruang $\mathcal{F}$-metrik; pemetaan kalis tukar tertib; titik tetap sepunya; Pemetaan pengecutan Boyd-Wong; Pemetaan pengecutan Meir-Keeler

References

Alnaser L.A., Lateef D., Fouad H.A. & Ahmad J. 2019. Relation theoretic contraction results in $\mathcal{F}$-metric spaces. *J. Nonlinear Sci. Appl.* **12**(5): 337–344.

Altun I. & Erduran A. 2022. Two fixed point results on $\mathcal{F}$-metric spaces. *Topology Algebra Appl.* **10**(1): 61–67.

Asif A., Nazam M., Arshad M. & Kim S.O. 2019. $\mathcal{F}$-metric, $\mathcal{F}$-contraction and common fixed-point theorems with applications. *Mathematics* **7**(7): 586.

Bera A., Dey L., Som S., Garai H. & Sintunavarat W. 2022. Boyd-Wong contractions in $\mathcal{F}$-metric spaces and applications. *Applied General Topology* **23**(1): 157–167.

Bera A., Garai H., Damjanovic B. & Chanda A. 2019. Some interesting result on $\mathcal{F}$-metric spaces. *Filomat* **33**(10): 3257–3268.

Binbasioglu D. 2021. Some result of common fixed point for compatible mappings $\mathcal{F}$-metric spaces. *Journal of New Theory* **37**: 1–7.

Boyd D.W. & Wong J.S.W. 1969. On nonlinear contraction. *Proc. Amer. Math. Soc.* $\mathbf{20}(2)$, pp. 458–464.

Cho Y.J., Murthy P. & Jungck G. 1993. A common fixed point theorem of Meir and Keeler type. *Internat. J. Math. Math. Sci.* **16**(4): 669–674.

Geraghty M.A. 1973. On contractive mappings. *Proc. Amer. Math. Soc.* $\mathbf{40}(2)$, pp. 604–608.

Gopal D. & Bisht R.K. 2017. Metrical common fixed points and commuting type mappings. In Gopal D., Kumam, P. & Abbas M. (eds.). *Background and Recent Development of Metric Fixed Point Theory*: 29-67. Boca Raton, FL: CRC Press.

Gopal D., Patel D.K. & Shukla S. 2017. Banach fixed point theorem and its generalizations. In Gopal D., Kumam, P. & Abbas M. (eds.). *Background and Recent Development of Metric Fixed Point Theory*: 1-28. Boca Raton, FL: CRC Press.

Jachymski J.R. 1997. Equivalence of some contractivity properties over metrical structures. *Proc. Amer. Math. Soc.* $\mathbf{125}(8)$, pp. 2327–2335.

Jahangir F., Haghmaram P. & Nourouzi K. 2021. A note on $\mathcal{F}$-metric spaces. *Journal of Fixed Point Theory and Applications* **23**(1): 2.

Jleli M. & Samet B. 2018. On a new generalization of metric spaces. *Journal of Fixed Point Theory and Applications* **20**(3): 128.

Jungck G. 1976. Commuting mappings and fixed points. *Amer. Math. Mon.* **83**(4): 261–263.

Karapınar E. & Agarwal R.P. 2022. *Fixed Point Theory in Generalized Metric Spaces*. Springer Nature.

Manav N. & Turkoglu D. 2019. Common fixed point results on modular $\mathcal{F}$-metric spaces. *AIP Conf. Proc.* $\mathbf{2183}$.

Matkowski J. 1994. Nonlinear contraction in metrically convex space. *Publ. Math. Debrecen* **45**(12): 103–114.

Meir A. & Keeler E. 1969. A theorem on contraction mappings. *J. Math. Anal. Appl.* **28**: 326–329.

Mitrović Z.D., Aydi H., Hussain N. & Mukheimer A. 2019. Reich, Jungck, and Berinde common fixed point results on $\mathcal{F}$-metric spaces and an application. *Mathematics* **7**(5): 387.

Park S. 1978. Fixed points of $\mathcal{F}$-contractive maps. *The Rocky Mountain Journal of Mathematics* **8**(4): 743–750.

Park S. & Bae J.S. 1981. Extensions of a fixed point theorem of Meir and Keeler. *Ark. Mat.* **19**(1-2): 223–228.

Som S., Bera A. & Dey L.K. 2020. Some remarks on the metrizability of $\mathcal{F}$-metric spaces. *J. Fixed Point Theory Appl.* **22**(1): 17.

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