

## CARREAU HYBRID NANOFILM FLOW OVER A STRETCHING SHEET WITH SUCTION/INJECTION EFFECT

(Aliran Nanofilem Carreau Hibrid di Atas Lembaran Regangan dengan Kesan Sedutan/Suntikan)

DIAN ADLINE JALALUDDIN\*, ROSLINDA NAZAR, KOHILAVANI NAGANTHRAN & ISHAK HASHIM

### ABSTRACT

The current work investigates the thin liquid film flow problem in the Carreau hybrid nanofilm ( $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ ) on a stretching sheet numerically. There is a suction/injection effect at the stretching sheet's surface, which is permeable. The boundary value problem solver (bvp4c) in MATLAB is applied to solve the system of linear equations obtained from the partial differential equations by employing suitable similarity variables. The effect of suction was found to contribute to improving heat transfer performance. The suction parameters had a more significant impact on the skin friction coefficient and heat transfer rate than the injection parameters. It was determined that the diminution of the film thickness at the free surface and the improvement of convective heat transfer rates were the most significant effects of the injection.

*Keywords:* thin film flow; hybrid nanofilm; Carreau fluid

### ABSTRAK

Kajian ini mengkaji tentang masalah aliran filem cecair nipis dalam nanofilem nipis Carreau ( $\text{Al}_2\text{O}_3\text{-Cu}/\text{H}_2\text{O}$ ) pada helaian regangan secara berangka. Terdapat kesan sedutan/suntikan pada permukaan helaian regangan yang telap. Penyelesaian masalah nilai sempadan (bvp4c) dalam MATLAB digunakan untuk menyelesaikan sistem persamaan linear yang diperoleh daripada persamaan pembezaan separa dengan menggunakan pembolehubah persamaan yang sesuai. Kesan sedutan didapati menyebabkan peningkatan prestasi pemindahan haba yang lebih baik. Parameter sedutan mempunyai kesan yang lebih ketara ke atas pekali geseran kulit dan kadar pemindahan haba daripada parameter suntikan. Didapati juga bahawa pengurangan ketebalan filem pada permukaan bebas dan peningkatan kadar pemindahan haba perolakan terhadap kesan suntikan adalah yang paling ketara.

*Kata kunci:* aliran filem nipis; nanofilem hibrid; cecair Carreau

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*Department of Mathematical Sciences  
Faculty of Science and Technology  
Universiti Kebangsaan Malaysia  
43600 UKM Bangi  
Selangor DE, MALAYSIA  
E-mail: p109938@siswa.ukm.edu.my\*, rmn@ukm.edu.my, ishak\_h@ukm.edu.my*

*Institute of Mathematical Sciences  
Center for Data Analytics Consultancy & Services  
Faculty of Science  
Universiti Malaya  
50603 Kuala Lumpur, MALAYSIA  
E-mail: kohi@um.edu.my*

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\*Corresponding author