HYPERSTABILITY RESULTS FOR THE GENERAL LINEAR FUNCTIONAL EQUATION IN NON-ARCHIMEDEAN 2-BANACH SPACES
(Hiperstabil bagi Persamaan Fungsi Linear Am di dalam Ruang Bukan Archimedean 2- Banach)

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ABSTRACT

Let X be a 2-normed space over $\mathbb{R}$, Y be a non-Archimedean 2-Banach space over non-Archimedean field $\mathbb{K}$, $r, s \in \mathbb{R} \setminus \{0\}$, and $R, S \in \mathbb{K} \setminus \{0\}$. In this paper, a short preface on non-Archimedean 2-Banach spaces $(Y, \|\cdot,\cdot\|_*)$ is given. Then, we reformulate the Brzdek fixed point theorem in non-Archimedean 2-Banach spaces. Using the Brzdek fixed point method, we prove hyperstability results of the general linear functional equation $h(rx + sy) = Rh(x) + Sh(y)$, $x, y \in X$, in non-Archimedean 2-Banach spaces. In fact, under some natural assumptions on control function $\gamma: X \times X \times Y \to [0, \infty)$, we show that every map satisfying $\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq \gamma(x, y, z)$, $x, y \in X$, $z \in Y$, is hyperstable in the class of functions $h: X \to Y$.

Keywords: non-Archimedean 2-Banach spaces; general linear functional equation; hyperstability; fixed point method

ABSTRAK

Biarkan X menjadi ruang nyata 2-norma di atas $\mathbb{R}$, Y menjadi ruang bukan Archimedean 2-norma di atas $\mathbb{K}$, $r, s \in \mathbb{R} \setminus \{0\}$, dan $R, S \in \mathbb{K} \setminus \{0\}$. Dalam penyelidikan ini, ringkasan mengenai ruang bukan Archimedean 2-Banach $(Y, \|\cdot,\cdot\|_*)$ diberikan. Kemudian, kami merumuskan semula teorem titik tetap Brzdek di dalam ruang bukan-Archimedean 2- Banach. Menggunakan kaedah titik tetap Brzdek, kami membuktikan ciri-ciri hiperstabil bagi persamaan fungsi linear am $h(rx + sy) = Rh(x) + Sh(y)$, $x, y \in X$, di dalam ruang bukan Archimedean 2-Banach. Malah, di bawah beberapa andaian pada fungsi kawalan $\gamma: X \times X \times Y \to [0, \infty)$, kami menunjukkan bahawa setiap pemetaan yang memenuhi kondisi $\|h(rx + sy) - Rh(x) - Sh(y), z\|_* \leq \gamma(x, y, z)$, $x, y \in X$, $z \in Y$, adalah hiperstabil di dalam fungsi kelas $h: X \to Y$.

Kata kunci: ruang bukan Archimedean 2-Banach; persamaan fungsi linear am; hiperstabil; kaedah titik tetap

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