INTERVAL ESTIMATION FOR PARAMETERS OF A BATHTUB HAZARD MODEL
(Penganggaran Selang untuk Parameter Model Hazad Tab Mandi)

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ABSTRACT

In this study, a two-parameter lifetime model has been extended to incorporate covariate in the presence of right-censored data. The model has bathtub-shaped or increasing failure rate function which enables it to fit real lifetime data set. The method of maximum likelihood was used to estimate the parameters in the model and a simulation study was then conducted to evaluate the performance of parameter estimates at various sample sizes and censoring proportion levels. The results from simulation study show that larger sample sizes and smaller censoring proportion give better estimates. Further, two interval estimation methods: Wald and likelihood ratio were constructed, and the performance of these methods was evaluated based on a coverage probability study. Both Wald and likelihood ratio techniques appear to have better performance when the sample size is larger. Also, a real right-censored lifetime data on patients with multiple myeloma was employed to illustrate the practical application of the extended model.

Keywords: bathtub-shaped; interval estimation; likelihood ratio; Wald; coverage probability study.

ABSTRAK

Dalam kajian ini, model jangka hayat yang mempunyai dua parameter telah dikembangkan dengan memasukkan kovariat dengan data yang tertapis kanan. Model tersebut mempunyai fungsi kadar kegagalan yang menaik ataupun berbentuk seperti tab mandi yang membolehkan model ini disesuaikan dengan data jangka hayat yang sebenar. Kaedah kebolehjadian maksimum digunakan untuk membuat anggaran parameter di dalam model ini dan seterusnya kajian simulasi dijalankan untuk menilai prestasi anggaran parameter pada beberapa saiz sampel dan kadar tapisan yang berlainan. Hasil kajian simulasi itu menunjukkan saiz sampel yang besar dan kadar tapisan yang kecil menghasilkan anggaran yang lebih baik. Selanjutnya, dua kaedah anggaran selang: Wald dan likelihood ratio telah dibina dan prestasi setiap kaedah tersebut dinilai melalui kajian kebarangkalian liputan. Kedua-dua kaedah Wald dan likelihood ratio dilihat mempunyai prestasi yang lebih baik apabila saiz sampel lebih besar. Data sebenar iaitu data jangka hayat tertapis kanan yang melibatkan pesakit myeloma pelbagai juga digunakan untuk menunjukkan aplikasi model yang telah dikembangkan ini secara praktiknya.

Kata kunci: berbentuk tab mandi; selang keyakinan; butstrap; likelihood ratio; Wald; kaedah kebarangkalian liputan.

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