

CLASSIFICATION OF FIXED POINTS OF POTTS–BETHE MAPPING OF DEGREE FOUR ON \mathbb{Q}_5

(Pengelasan Titik Tetap Pemetaan Potts–Bethe Darjah Empat pada \mathbb{Q}_5)

MOHD ALI KHAMEINI AHMAD*, MURAT ALP, MOHAMMAD AZIM MOHD AZAHARI & AHMAD FADILLAH EMBONG

ABSTRACT

The Potts–Bethe mapping is a rational function arises in the study of the Potts model on the Cayley tree (or Bethe lattice). In this paper, the Potts–Bethe mapping of degree four is considered over the field \mathbb{Q}_5 of 5-adic numbers. In some regimes (a condition appear in the study of p -adic Potts model), the fixed points are found and their stability are determined. It is done by solving some quartic equation over \mathbb{Q}_5 and calculating the value of derivative at each fixed points. This is the continuation of the previous work where contraction and chaos are found, but here other property is realized such as 1-Lipschitz.

Keywords: rational function; p -adic number; fixed point

ABSTRAK

Pemetaan Potts–Bethe ialah fungsi nisbah yang timbul dalam kajian model Potts pada pokok Cayley (atau kekisi Bethe). Dalam artikel ini, pemetaan Potts–Bethe darjah empat dipertimbangkan di atas medan \mathbb{Q}_5 bagi nombor 5-adic. Dalam sesetengah rejim (keadaan yang muncul dalam kajian model Potts p -adic), titik tetap ditemui dan tingkah lakunya dikelaskan. Ia dilakukan dengan menyelesaikan persamaan kuartik pada \mathbb{Q}_5 dan mengira nilai pembezaan di setiap titik. Ini adalah kesinambungan kerja sebelumnya di mana pengecutan dan kekacauan ditemui, tetapi di sini sifat lain ditemui seperti 1-Lipschitz.

Kata kunci: fungsi nisbah; nombor p -adic; titik tetap

References

- Ahmad M.A.K., Liao L. & Saburov M. 2018. Periodic p -adic Gibbs measures of q -states Potts model on Cayley trees I: The chaos implies the vastness of the set of p -adic Gibbs measures. *J. Stat. Phys.* **171**(6): 1000–1034.
- Ahmad M.A.K., Pah C.H. & Saburov M. 2019. Dynamics of Potts–Bethe mapping of degree four on \mathbb{Q}_5 . *AIP Conference Proceeding*, 2184, p. 020001.
- Fan A.H., Fan S.L., Liao L.M. & Wang Y.F. 2014. On minimal decomposition of p -adic homographic dynamical systems. *Adv. in Math.* **257**: 92 – 135.
- Fan A.H., Fan S.L., Liao L.M. & Wang Y.F. 2017. Minimality of p -adic rational maps with good reduction. *Discrete Contin. Dyn. Syst. Ser. A* **37**(6): 3161–3182.
- Fan A.H., Liao L.M., Wang Y.F. & Zhou D. 2007. p -adic repeller in \mathbb{Q}_p are subshifts of finite type. *Comptes Rendus Mathematique* **344**(4): 219–224.
- Fan S.L. & Liao L.M. 2018. Rational map $ax + 1/x$ on the projective line \mathbb{Q}_2 . *Sci. China Math.* **61**(12): 2221–2236.
- Khakimov O. & Mukhamedov F. 2022. Chaotic behavior of the p -adic Potts–Bethe mapping II. *Ergod. Th. & Dynam. Sys.* **42**: 3433–3457.
- Mukhamedov F. & Khakimov O. 2016. Phase transition and chaos: p -adic potts model on a Cayley tree. *Chaos Solitons and Fractals* **87**: 190–196.
- Mukhamedov F. & Khakimov O. 2018. Chaotic behavior of the p -adic Potts–Bethe mapping. *Discrete Contin. Dyn. Syst. Ser. A* **38**(1): 231–245.

- Mukhamedov F. & Khakimov O. 2021. Translation-invariant generalized p -adic Gibbs measures for the Ising model on Cayley trees. *Math. Methods Appl. Sci.* **44**(16): 12302–12316.
- Mukhamedov F. & Khakimov O. 2023. p -adic phase transitions for countable state Potts model. *Math. Methods Appl. Sci.* **46**(13): 14104–14119.
- Mukhamedov F., Omirov B. & Saburov M. 2014. On cubic equations over p -adic field. *Int. J. Number Theory* **10**(5): 1171–1190.
- Mukhamedov F., Rahmatullaev M.M., Tukhtabaev A.M. & Mamadjonov R. 2023. The p -adic Ising model in an external field on a Cayley tree: periodic Gibbs measures. *Theor. Math. Phys.* **216**(2): 1238–1253.
- Rahmatullaev M. & Tukhtabaev A. 2023. Some non-periodic p -adic generalized Gibbs measures for the Ising model on a Cayley tree of order k . *Math. Phys. Anal. Geom.* **26**: 22.
- Rozikov U.A., Sattarov I.A. & Tukhtabaev A.M. 2022. Periodic points of a p -adic operator and their p -adic Gibbs measures. *P-Adic Num. Ultramet. Anal. Appl.* **14**(Suppl 1): S30–S44.
- Saburov M. & Ahmad M.A.K. 2015a. On descriptions of all translation invariant p -adic Gibbs measures for the Potts model on the Cayley tree of order three. *Math. Phys. Anal. Geom.* **18**: 1–33.
- Saburov M. & Ahmad M.A.K. 2015b. Quadratic equations over p -adic fields and their application in statistical mechanics. *ScienceAsia* **41**(3): 209–215.
- Saburov M. & Ahmad M.A.K. 2018. Local descriptions of roots of cubic equations over p -adic fields. *Bull. Malays. Math. Sci. Soc.* **41**(2): 965–984.
- Saburov M., Ahmad M.A.K. & Alp M. 2021. The study on general cubic equations over p -adic fields. *Filomat* **35**(4): 1115–1131.

Department of Mathematical Sciences

Faculty of Science

Universiti Teknologi Malaysia

81310 Johor Bahru

Johor; MALAYSIA

E-mail: ahmadfadillah@utm.my, khameini.ahmad@gmail.com, ma.khameini@utm.my,*

mohammadazim@graduate.utm.my,

Mathematics Department

College of Engineering and Technology

American University of the Middle East

Egaila, Kuwait

E-mail: murat.alp@aum.edu.kw

Received: 16 August 2023

Accepted: 7 March 2024

*Corresponding author