ON THE RELATION OF SEVEN CHAOS CHARACTERIZATIONS
(Hubungan antara Tujuh Ciri Kekalutan)

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ABSTRACT

In this work, we explore seven chaos-related notions in dynamical systems: locally everywhere onto, mixing, totally transitive, strong dense periodicity, blending, specification, and Devaney chaos. We analyze their interrelations, proving positive connections and providing counterexamples for negative ones. Our findings establish a hierarchy among these chaos characterizations, with the specification property at the top and blending, transitivity, and strong dense periodicity at the bottom in compact spaces. In shifts of finite type, these properties are equivalent, but this equivalence does not hold in shifts of infinite type.

Keywords: Devaney chaos; locally everywhere onto; totally transitivity; topologically mixing; strong dense periodic points; blending

ABSTRAK

Dalam kajian ini, kami meneroka tujuh konsep berkaitan kekalutan dalam sistem dinamik: sifat keseluruh setempat di mana-mana, pencampuran, transitif sepenuhnya, sifat berkalala tumpat yang kuat, pencampuran, spesifikasi, dan kekalutan Devaney. Kami menganalisis hubungan antara konsep-konsep ini dengan membuktikan hubungan positif dan memberikan contoh penyangkal untuk hubungan negatif. Penemuan kami menunjukkan hierarki antara ciri-ciri kekacauan ini, dengan sifat spesifikasi berada di puncak, manakala sifat pencampuran, transitiviti, dan sifat berkala tumpat yang kuat berada di kedudukan terendah dalam ruang padat. Dalam ruang anjakan jenis terhingga, sifat-sifat ini adalah setara, tetapi kesetaraan ini tidak berlaku dalam anjakan jenis tidak terhingga.

Kata kunci: kekalutan Devaney; keseluruh setempat di mana-mana; transitif sepenuhnya; pencampuran secara bertopologi; berkala tumpat yang kuat; pencampuran

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