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PREMIUMS DIVERSITY OF GROUP PENSION FUND BASED ON THE AGGREGATE COST METHOD

(Kepelbagaian Premium Kumpulan Wang Pencen Berdasarkan Kaedah Kos Agregat)

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ABSTRACT

Participants can withdraw pension fund insurance before reaching the normal retirement age due to factors such as death, permanent disability, or early retirement. As a result, the participant's premium payment differs from what they would pay in retirement. The aggregate cost method is one of the methods used to calculate pension contributions. The purpose of this study is to determine the current value of pension benefits and insurance premiums based on uniform assumptions using the aggregate cost method. This calculation method is based on common characteristics that are influenced by the amount of salary and the rate of increase of the insured salary while working. The results showed that male participants received more pension benefits than female participants, who received the same salary, salary increase rate, and entry age. Meanwhile, male participants paid higher premiums than female participants.

Keywords: aggregate cost method; benefits; premiums; present values; pension fund

ABSTRAK

Peserta insurans untuk insurans dana pencen boleh keluar sebelum mencapai umur persaraan biasa disebabkan faktor lain seperti kematian, hilang upaya kekal atau persaraan awal. Ini menyebabkan bayaran premium yang perlu dibayar oleh peserta berbeza dengan bayaran premium semasa persaraan biasa. Salah satu kaedah yang digunakan untuk menentukan jumlah caruman pencen ialah kaedah kos agregat. Tujuan kajian ini adalah untuk menentukan nilai semasa faedah pencen dan premium insurans berdasarkan andaian seragam menggunakan kaedah kos agregat. Kaedah pengiraan ini adalah berdasarkan kumpulan yang mempunyai ciri-ciri tertentu yang sama yang dipengaruhi oleh jumlah gaji dan kadar kenaikan gaji yang diinsuranskan semasa bekerja. Berdasarkan dapatan kajian daripada sampel lelaki dan wanita yang pergi kerana meninggal dunia, hilang upaya kekal, atau bersara awal, diketahui bahawa bagi peserta lelaki dan perempuan yang mempunyai gaji yang sama, kadar kenaikan gaji, dan umur kemasukan, nilai faedah pencen yang diterima oleh lelaki adalah lebih besar daripada yang diterima oleh wanita, dan premium yang dibayar oleh peserta lelaki adalah lebih tinggi daripada yang dibayar oleh peserta wanita.

Kata kunci: kaedah kos agregat; faedah; premium; nilai kini; dana pencen

1. Introduction

Employees at a specific company or institution are concerned about uncertainty, which could result in significant losses, particularly as they approach retirement age. A suitable option would be a pension fund programme or pension fund insurance. Pension fund insurance provides a sum of money to the participant upon reaching retirement age. This is in accordance with an agreement agreed upon by both parties, namely the insurance participant and the insurance company (Sibuea *et al*. 2014).

Pension fund insurance is one of the insurance products that is designed to offer policyholders assurances upon their retirement. Employees contribute to pension funds on a regular basis (Chen & Matkin 2017). The payment may be made on a monthly or annual basis. The pension fund will be managed and run by an institution to enable participants to obtain their retirement benefits in a timely manner or in full. There are several types of pensions, namely normal, disability, death (often called widow's or widower's) and retirement accelerated pensions (Apnesia Feronika 2019).

Sibuea *et al*. (2014) stated that the number of pension premiums paid by the insured is influenced by the benefit or money received. Pension premiums are obligations paid by the insured to the insurance company in accordance with the pension fund regulations. According to Futami (1993), the premium paid varies depending on the type of insurance and annuity. A lifetime annuity is used to calculate the amount of a life annuity pension premium. Dickson *et al.* (2009) proposed that one method for determining pension fund insurance participants' life chances and withdrawal possibilities is to employ uniform assumptions. These uniform assumptions posit that the withdrawal possibilities for participants' insurance remain constant across time.

The cost aggregate method is one of the methods used to determine the number of pension premiums. This method is based on a group with a certain characteristic equation that shows the normal contribution rate or the premium payable depending on the level of the financing actuarial obligation at a certain time, which indicates the value of pension benefits based on past services up to a predetermined time (Futami 1994).

Sibuea *et al*. (2014) employed the aggregate cost method to calculate the annual insurance premium of the pension fund for ordinary pension cases. A pension fund is a type of insurance that helps people save for their retirement. Sometimes, people may leave the pension fund before they reach the normal retirement age because of death, permanent disability, or early retirement. This causes the premiums payable by the insured to the pension fund to be different from the payment of ordinary pension premiums.

This study examines the suitability of the cost aggregate method for determining the value of pension contributions to a group pension fund in Indonesia. The value of pension benefits and premiums in three distinct retirement cases is evaluated and compared. The aggregate cost method calculates the premium amount based on the average total salary over the term of employment. If inflation occurs,this method does not change premiums.

2. Methodology

2.1. *Uniform assumptions for multiple decrement cases*

A uniform assumption is an assumption that states the chances of withdrawal for an insurer at any time are the same. According to Dickson *et al.* (2009), the chances of withdrawal at aged x years to t the following year at intervals $0 \leq t < 1$ that are annotated with ${}_tq_x$ using uniform assumptions are expressed as follows:

$$ {}_tq_x^{(j)} = t\, q_x^{(j)} \qquad (1)$$

$J(x) = j$ which are cases that cause customer reduction $j = 1, 2, \ldots, m$ and

$$ {}_tq_x^{(T)} = t\, q_x^{(T)} \qquad (2)$$

$T(x) = T$ combined total exit opportunities for $j = 1, 2, \ldots, m.$

For example, the 1999 Indonesian Mortality Table for Women data revealed that the participant's age when purchasing a one-year insurance was at the age of 21 years old. From this data, the uniform assumptions are calculated as follows:

$x = 21$ year

$t\ = 1$ year

$_{t}q_x = \frac{L_x - L_{x+t}}{L_x}$

$_{1}q_{21} = \frac{L_{21} - L_{22}}{L_{21}} = \left(1 - \frac{L_{22}}{L_{21}}\right) = \left(1 - \frac{98339}{98427}\right) = 0.00089$

$tq_x = 1\ \cdot\ q_{21} = 1\ \cdot\ 0.00089 = 0.00089$

proven that $_{t}q_x = tq_x$.

2.2. *Bability of life and exit for one case and three cases*

The declining number of clients is common in insurance, especially in pension fund insurance (Bowers *et al.* 1997). This decline will affect the participant's chances of survival and termination. The decline of clients caused by one case is called a single reduction, while the cause of the decline of more than three cases is called a double reduction.

x is the age of a participant purchasing the insurance and the time of deduction denoted by $T(x) = T$, that is, the time when x terminates an insurance. Furthermore, it is assumed that there are m cases that lead to decline. Also, given $J(x) = j$, which is a case that causes a reduction in the client where, $j = 1, 2, \dots, m$.

$f_{T(x),J(x)}(t, j)$ represents a combination of probability distribution functions for $T(x)$ and $J(x)$ (Finan 2011). This combination of probability distribution functions can be used to calculate the probability of an event described by $T(x)$ and $J(x)$. Finan (2011) stated that the probability of withdrawal for a double reduction case is defined by:

$$q_x^j = \int_0^t f_{T(x),J(x)}(s, j)ds \tag{3}$$

2.3. *Mortality acceleration*

The probability of withdrawal for one case and three cases is influenced by the acceleration of death. According to Bowers *et al.* (1997), an acceleration of death for case *j* has specified

$$\mu_x^{(j)}(t) = \frac{f_{T(x),J(x)}(t,j)}{p_x^T} \tag{4}$$

Based on Eqs. (3) and (4), the acceleration of mortality for the case of *j* can be written as follows:

$$\mu_x^{(j)}(t) = \frac{1}{p_x^T}\frac{d}{dt}q_x^j \tag{5}$$

and the total acceleration of death for all cases, namely:

$$\mu_x^{(T)}(t) = \frac{1}{p_x^T}\frac{d}{dt}q_x^{(T)} \tag{6}$$

Based on Eq. (4), it is also obtained:

$$f_{T(x),j(x)}(t,j) = p_x^T \mu_x^{(j)}(t) \tag{7}$$

Based on Eqs. (3) and (7) the probability of exit for one case can be written as:

$$q_x^{1(j)} = \int_0^t p_x^{i(j)}\, \mu_x^{(j)}(t)\ dt \tag{8}$$

Furthermore, the relationship between total survival chances and survival chances with one case associated with the acceleration of death is:

$$p_x^{(T)} = \exp\left\{-\int_0^t \left[\mu_x^{(1)}(s) + \mu_x^{(2)}(s) + \cdots + \mu_x^{(m)}(s)\right] ds\right\}$$

$$p_x^{(T)} = \prod_{j=1}^m \exp\left\{-\int_0^t \mu_x^{(j)}(s)\ ds\right\}$$

$$p_x^{(T)} = \prod_{j=1}^m p_x^{i(j)} \tag{9}$$

Bowers *et al.* (1997) stated that the probability of withdrawal for three cases is as follows:

$$q_x^{(j)} = \int_0^t p_x^{(T)}\, \mu_x^{(j)}(t)dt \tag{10}$$

Then, from Eqs. (8), (9) and (10), it is known that $q_x^{1(j)} \geq q_x^{(j)}$.

This study assumes that there are three cases that cause early retirement. So the chances of withdrawal for multiple decrement cases (death, permanent disability and early retirement) are assuming uniformity. The withdrawal odds for the three cases are as follows:

- Case I: $j = 1$ for death, based on Eq. (10) and the equation of a participant's withdrawal opportunity for the death case is as follows:

$$q_x^{(1)} = \int_0^1 {}_tp_x^{(T)}\mu_x^{(1)}(t)dt$$

$$q_x^{(1)} = q_x^{1(1)}\left(1 - \frac{1}{2}q_x^{1(2)} - \frac{1}{2}q_x^{1(3)} + \frac{1}{3}q_x^{1(2)}q_x^{1(3)}\right) \tag{11}$$

- Case II: $j = 2$ for permanent disability , done the same way then the equation of the participant's changes exit for a case of permanent disability expressed as follows:

$$q_x^{(2)} = \int_0^1 {}_tp_x^{(T)}\mu_x^{(2)}(t)dt$$

$$q_x^{(2)} = q_x^{1(2)}\left(1 - \frac{1}{2}q_x^{1(1)} - \frac{1}{2}q_x^{1(3)} + \frac{1}{3}q_x^{1(1)}q_x^{1(3)}\right) \tag{12}$$

- Case III: Early retirement is a person's decision to stop working before reaching the official retirement age, which is usually determined by law or company regulations. This choice can be influenced by various personal factors, such as health conditions, life plans, financial goals, and the desire to pursue interests outside the scope of work. The early retirement case expressed by $j = 3$ is done in the same way as in the case ($j = 1$) and ($j = 2$) so that the withdrawal opportunity is obtained as follows:

$$q_x^{(3)} = \int_0^1 \ {}_t p_x^{(T)} \mu_x^{(3)}(t) dt$$

$$q_x^{(3)} = q_x^{1(3)} \left(1 - \frac{1}{2} q_x^{1(1)} - \frac{1}{2} q_x^{1(2)} + \frac{1}{3} q_x^{1(1)} q_x^{1(2)}\right) \tag{13}$$

2.4. *Annuity*

In determining the amount of life annuity pension premium uses a lifetime annuity. According to Futami (1994), early lifetime annuities for three cases are expressed as follows:

$$\ddot{a}_x = \sum_{t=0}^{\omega-x-1} v^t \, p_x^{(T)} \tag{14}$$

where, ω is the estimated maximum age of the insurance participant, x is the age of the insured, $p_x^{(T)}$ is the sum of the probability of life of the insured of the pension fund aged x years and lasting up to the age of $x + t$ years, and the discount factor denoted by v as follows:

$$v = \frac{1}{1+i} \tag{15}$$

where i is the interest rate.

2.5. *Aggregate cost method*

The aggregate cost method is a method in which the calculations are based on groupings with certain characteristics in common, namely indicating the level of normal contributions or premiums to be paid depending on the level of actuarial liability financing at a certain time. In this study, participants were grouped into three cases: normal retirement, permanent disability, and death, with the assumption being that the withdrawal probability for each participant is the same. Therefore, this method is suitable for this study as compared to other methods. According to Futami (1994), the salary of the pension fund insured at the age *x* year for the following *t* years is as follows:

$$c_{x+t} = c_x (1 + z)^t \tag{16}$$

c_{x+t} : Total salary of pension fund insurance participants at age *x* years for the following *t* years
c_x : Total salary of pension fund insurance participants at age *x* years
z : The level of annual salary increase provided by the company

where, c_x is the salary of the insured netto at the age *x* of entry year and *z* represents the rate of increase in the annual salary provided by the company. Let r be the normal retirement age of the participant of the pension fund and the percentage of the value of defined benefits determined by the insurer is expressed by *k*, then the value of the defined benefit for the pension fund is as follows:

$$B_r = k(r - x)C_{r-1} \tag{17}$$

x : the age of entry to work for pension fund insurance participants
r : the normal retirement age of pension fund participants
k : the percentage of pension benefit value determined by the insurance company
C_{r-1} : the total salary of pension fund insurance participants in the last year of work, namely one year before entering normal retirement age

After obtaining the value of the pension benefit, we can determine the present value of the pension benefit. The present value of the pension benefit can be determined as follows:

$$\tilde{A}_x = B_r p_x^{(T)} v^{r-x} \ddot{a}_r \tag{18}$$

$\tilde{A}_x$: Present value of participant's retirement benefits at age x years
B_r : The value of pension benefits that retire at age r years
$p_x^{(T)}$: Total life chances of pension fund insurance participants who are x years old
x : Age of insurance participant
v : Faktor diskon
$\ddot{a}_r$: Cash value of initial lifetime annuity for age r years

The amount of premiums that the insured of the pension fund must pay at the age of *x*, which is denoted by P_x, F states the amount of pension funds provided by the company to pension fund insurance participants. Then, in general, the formula of pension premiums of the pension fund insurance group using the method of aggregate cost is as follows:

$$P_x = \frac{\tilde{A}_x - F}{\ddot{a}_r} \tag{19}$$

3. Results and Discussions

Premiums are paid by participants who terminate pension fund insurance (or provide pension funds for employees) in addition to the normal age pension due to death, permanent disability, and early retirement. can be seen in Eqs. (11), (12) and (13).

3.1. *Early lifetime annuity with uniform assumptions for three cases*

After obtaining the probability of withdrawal for three cases using a uniform assumption, and based on Eq. (14), the initial lifetime annuity equation of three cases can be calculated as

$$\ddot{a}_r = \Big[\sum_{t=0}^{\omega-r-1} v^t - v^t t\, q_r^{1(1)} - v^t t q_r^{1(2)} - v^t t q_r^{1(3)} + v^t t q_r^{1(1)} q_r^{1(2)} + v^t t q_r^{1(1)} q_r^{1(3)} + v^t t q_r^{1(2)} q_r^{1(3)} - v^t t q_r^{1(1)} q_r^{1(2)} q_r^{1(3)}\Big] \tag{20}$$

3.2. *Present value benefit of pension fund insurance participants*

After obtaining the value of the initial lifetime annuity for the case of double depreciation, by substituting Eqs. (9), (14) and (17) into Eq. (18) the present value equation for pension benefits for double depreciation cases is as follows:

$$\tilde{A}_x = (k(r-x)C_{r-1})p_x^{(T)}v^{r-x}\left[\sum_{t=0}^{\omega-x-1}[v^t - v^t t\, q_r^{1(1)} - v^t tq_r^{1(2)} - v^t tq_r^{1(3)} + v^t tq_r^{1(1)}q_r^{1(2)} + v^t tq_r^{1(1)}q_r^{1(3)} + v^t tq_r^{1(2)}q_r^{1(3)} - v^t tq_r^{1(1)}q_r^{1(2)}q_r^{1(3)}\right] \tag{21}$$

3.3. *Pension premiums for three cases using the cost aggregate method*

After obtaining the equations for the value of the initial lifetime annuity and the present value of the pension benefit, based on Eq. (19), the general formula for pension fund insurance premiums using the cost aggregate method is as follows:

$$P_x = \frac{\tilde{A}_x - F}{\ddot{a}_r}$$

$$= \frac{(k(r-x)C_{r-1})p_x^{(T)}v^{r-x}\begin{bmatrix}\sum_{t=0}^{\omega-x-1}[v^t - v^t tq_r^{1(1)} - v^t \\ tq_r^{1(2)} - v^t tq_r^{1(3)} + v^t\ tq_r^{1(1)} \\ q_r^{1(2)} + v^t tq_r^{1(1)}q_r^{1(3)} + v^t t \\ q_r^{1(2)}q_r^{1(3)} + v^t tq_r^{1(1)}q_r^{1(2)}q_r^{1(3)}]\end{bmatrix} - F}{\begin{bmatrix}\sum_{t=0}^{\omega-x-1}[v^t - v^t tq_r^{1(1)} - v^t \\ tq_r^{1(2)} - v^t tq_r^{1(3)} + v^t\ tq_r^{1(1)} \\ q_r^{1(2)} + v^t tq_r^{1(1)}q_r^{1(3)} + v^t t \\ q_r^{1(2)}q_r^{1(3)} + v^t tq_r^{1(1)}q_r^{1(2)}q_r^{1(3)}]\end{bmatrix}} \tag{22}$$

Based on Law No. 40/2004, concerning the National Social Security System (SJSN) of Indonesia, it stipulates that the first retirement age is 56 years. For example, a man and a woman who become employees at a company and join a pension fund insurance at the age of 23. The retirement age set by the government is 56 years old. It is assumed that the basic salary received by the employee in the first month of work is Rp 1,241,250.00 per month with a salary increase of 5% per annum and the value of pension benefits provided by the company to the insured of the pension fund is 2.5%. In February 2020, the man was seriously ill, causing death, while the woman had a fatal accident that resulted in permanent disability. The insurance company calculated the final premium that both employees would have to pay and the number of current retirement benefits the insurance company would have to pay.

(1) The first step was to calculate the present value of the benefit. The pension fund's premise was to determine the magnitude of the salary increase from the start of the work year to one year before entering the normal pension period. The first-year compensation for male and female employees is calculated by multiplying the baseline monthly salary by 12 months, resulting in Rp 1,241,250.00 × 12 = Rp 14,895,000.00. Based on Eq. (11), the one-year salary for men and women is c_{24} = Rp 15,639,750.00 up to c_{55} = Rp 70,973,803.17

Table 1: Data on salary increases of male and female participants of age 24 years old to age 55

t	x	$(1+z)^t$	$c_{x+t} = c_x(1+z)^t$ (Rp)
1	24	1.05	15,639,750.00
2	25	1.10	16,421,737.50
3	26	1.16	17,242,824.38
4	27	1.22	18,104,956.38
5	28	1.28	19,010,213.87
6	29	1.34	19,960,724.57
7	30	1.40	20,958,760.80
8	31	1.48	22,006,698.84
9	32	1.55	23,107,033.78
10	33	1.63	24,262,385.47
11	34	1.71	25,475,504.74
12	35	1.80	26,749,279.98
13	36	1.89	28,086,743.97
14	37	1.98	29,491,081.17
15	38	2.08	30,965,635.23
16	39	2.18	32,313,916.99
17	40	2.29	34,139,612.84
18	41	2.40	35,846,593.49
19	42	2.52	37,638,923.16
20	43	2.65	39,520,869.32
21	44	2.79	41,496,912.78
22	43	2.93	43,571,758.42
23	46	3.07	45,750,346.34
24	47	3.22	48,037,863.66
25	48	3.39	50,439,756.84
26	49	3.55	52,961,744.69
27	50	3.73	55,609,831.92
28	51	3.92	58,390,323.52
29	52	4.11	61,309,839.69
30	53	4.32	64,375,331.68
31	54	4.53	67,594,098.26
32	55	4.76	70,973,803.17

(2) Table 1 presents the initial income received by men and women at the age of 24, up until the final salary earned by the 55-year-old pension fund insurance participant, a year before entering normal retirement.

Based on Eq. (17), the amount of pension benefits with k = 2.5% using the amount of the last salary was as follows:

$$B_r = k(r-x)C_{r-1}$$

$$B_{56} = \text{Rp } 58{,}553{,}387.62$$

The next step was to determine the probability of each case terminate men and women. The probability of withdrawal at age $x = 23$ years, for the case of $j = 1$, namely:

$$q_{23}^{(1)} = q_{23}^{1(1)}\left(1 - \frac{1}{2}q_{23}^{1(2)} - \frac{1}{2}q_{23}^{1(3)} + \frac{1}{3}q_{23}^{1(2)}q_{23}^{1(3)}\right)$$

$$= 0.00186\left(1 - \left(\frac{1}{2} \times 0.00030\right) - \left(\frac{1}{2} \times 0\right) + \left(\frac{1}{3} \times 0.00030 \times 0\right)\right) = 0.00186$$

the great chance of going out for case $j = 2$ was as large as

$$q_{23}^{(2)} = q_{23}^{1(2)}\left(1 - \frac{1}{2}q_{23}^{1(1)} - \frac{1}{2}q_{23}^{1(3)} + \frac{1}{3}q_{23}^{1(1)}q_{23}^{1(3)}\right)$$

$$= 0.00030\left(1 - \left(\frac{1}{2} \times 0.00186\right) - \left(\frac{1}{2} \times 0\right) + \left(\frac{1}{3} \times 0.00186 \times 0\right)\right)$$

$$= 0.00030(1 - (0.00093) - 0 + 0) = 0.00030(0.99907)$$

$$= 0.00030.$$

the great chance of coming out for case $j = 3$ was

$$q_{23}^{(3)} = q_{23}^{1(3)}\left(1 - \frac{1}{2}q_{23}^{1(1)} - \frac{1}{2}q_{23}^{1(2)} + \frac{1}{3}q_{23}^{1(1)}q_{23}^{1(2)}\right)$$

$$= 0\left(1 - \left(\frac{1}{2} \times 0.00186\right) - \left(\frac{1}{2} \times 0.00030\right) + \left(\frac{1}{3} \times 0.00186 \times 0.00030\right)\right)$$

$$= 0(1 - (0.00093) - (0.00015) + 0)$$

$$= 0(0.99892) = 0.$$

and the total chance of withdrawal at the age of 23 was as large as

$$q_{23}^{(T)} = q_{23}^{(1)} + q_{23}^{(1)} + q_{23}^{(3)} = 0.00216.$$

for $q_{24}^{(1)}$, $q_{24}^{(2)}$, and $q_{24}^{(T)}$ from the age of 24 to the age of 56, $q_{23}^{(1)}, q_{23}^{(2)}, q_{23}^{(3)}$ and $q_{23}^{(T)}$ above.

Table 2: Mortality table of three cases for entry age x = 23 years

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$	$q_x^{(T)}$
23	0.00186	0.00030	0	0.00216
24	0.00182	0.00030	0	0.00212
25	0.00177	0.00030	0	0.00207
26	0.00173	0.00030	0	0.00203
27	0.00171	0.00030	0	0.00201
28	0.00170	0.00030	0	0.00200
29	0.00171	0.00030	0	0.00201
30	0.00173	0.00040	0	0.00213
31	0.00178	0.00040	0	0.00218
32	0.00183	0.00040	0	0.00223
33	0.00191	0.00040	0	0.00231
34	0.00200	0.00040	0	0.00240
35	0.00211	0.00040	0	0.00251
36	0.00224	0.00050	0	0.00274
37	0.00240	0.00060	0	0.00300
38	0.00258	0.00070	0	0.00328
39	0.00279	0.00080	0	0.00359
40	0.00302	0.00090	0	0.00392

Table 2 (Continued)

41	0.00329	0.00100	0	0.00429
42	0.00359	0.00120	0	0.00479
43	0.00387	0.00140	0	0.00526
44	0.00414	0.00160	0	0.00573
45	0.00455	0.00180	0	0.00634
46	0.00491	0.00199	0.00050	0.00741
47	0.00531	0.00219	0.00045	0.00795
48	0.00573	0.00249	0.00040	0.00862
49	0.00620	0.00279	0.00035	0.00934
50	0.00670	0.00309	0.00030	0.01009
51	0.00729	0.00339	0.00025	0.01092
52	0.00794	0.00378	0.00020	0.01193
53	0.00869	0.00418	0.00015	0.01302
54	0.00954	0.00458	0.00010	0.01421
55	0.01086	0.00493	0.00005	0.01585
56	0.01642	0.00496	0.00000	0.02138

(3) Before calculating the current pension benefits and the number of premiums, the next step was to calculate the initial lifetime annuity cash value for men and women. The normal retirement age determined by the company is 56 years old, and the estimated maximum age for men is 100 years old ($\omega_a = 100$), while the women's estimated maximum age f is 103 years old ($\omega_a = 100$). In Indonesia, the age limit for whole life insurance is set at a maximum age of 100 years old for men and 103 years for women. The interest rate used for calculating the value of the initial lifetime annuity was 7%. Based on Eq. (14), the number of early lifetime annuities for men (a) was as follows:

$$\ddot{a}_r(a) = \sum_{t=0}^{100-56-1}\Big[v^t - v^t t\, q_r^{1(1)} - v^t t q_r^{1(2)} - v^t t q_r^{1(3)} + v^t t q_r^{1(1)} q_r^{1(2)} + v^t t q_r^{1(1)} q_r^{1(3)} + v^t t q_r^{1(2)} q_r^{1(3)} - v^t t q_r^{1(1)} q_r^{1(2)} q_r^{1(3)}\Big]$$

$$\ddot{a}_{56}(a) = 10{,}80870923$$

Table 3: Early lifetime annuity of male participants for the age of 56 with an interest rate of 7%

t	v^t	$tp_{56}^{(T)}$	Annuity
0	1	1	1
1	0.9345794	0.97862	0.914598131
2	0.8734387	0.95724	0.836090488
3	0.8162979	0.93586	0.763940530
4	0.7628952	0.91448	0.697652413
5	0.7129862	0.89310	0.636767956
6	0.6663422	0.87172	0.580863842
7	0.6227497	0.85034	0.529549015
8	0.5820091	0.82896	0.482462266
9	0.5439337	0.80758	0.439270011
10	0.5083493	0.78620	0.399664212
11	0.4750928	0.76482	0.363360471
12	0.4440120	0.74344	0.330096250
13	0.4149644	0.72206	0.299629228

Table 3 (Continued)

14	0.3878172	0.70068	0.271735783
15	0.3624460	0.67930	0.246209580
16	0.3387346	0.65792	0.222860266
17	0.3165744	0.63654	0.201512262
18	0.2958639	0.61516	0.182003646
19	0.2765083	0.59378	0.164185117
20	0.2584190	0.57240	0.147919036
21	0.2415131	0.55102	0.133078540
22	0.2257132	0.52964	0.119546720
23	0.2109469	0.50826	0.107215862
24	0.1971466	0.48688	0.095986746
25	0.1842492	0.46550	0.085767992
26	0.1721955	0.44412	0.076475462
27	0.1609304	0.42274	0.068031703
28	0.1504022	0.40136	0.060365432
29	0.1405628	0.37998	0.053411058
30	0.1313671	0.3586	0.047108248
31	0.1227730	0.33722	0.041401513
32	0.1147411	0.31584	0.036239837
33	0.1072347	0.29446	0.031576329
34	0.1002193	0.27308	0.027367898
35	0.0936629	0.25170	0.023574962
36	0.0875355	0.23032	0.020161166
37	0.0818088	0.20894	0.017093139
38	0.0764569	0.18756	0.014340248
39	0.0714550	0.16618	0.011874393
40	0.0667804	0.14480	0.009669799
41	0.0624116	0.12342	0.007702836
42	0.0583286	0.10204	0.005951847
43	0.0545127	0.08066	0.004396993
	Total		10.80870923

Meanwhile, the number of initial lifetime annuities for women (b) is as follows:

$$\ddot{a}_r(b) = \sum_{t=0}^{103-56-1}[v^t - v^t t\, q_r^{1(1)} - v^t t q_r^{1(2)} - v^t t q_r^{1(3)} + v^t t q_r^{1(1)} q_r^{1(2)} + v^t t q_r^{1(1)} q_r^{1(3)} + v^t t q_r^{1(2)} q_r^{1(3)} - v^t t q_r^{1(1)} q_r^{1(2)} q_r^{1(3)}]$$

$$\ddot{a}_{56}(b) = 10.81426900$$

Table 4. Early lifetime annuities of female participants for age 56 with 7% interest rate

t	v^t	$tp_{56}^{(T)}$	Annuity
0	1	1	1
1	0.9345794	0.97862	0.914598131
2	0.8734387	0.95724	0.836090488
3	0.8162979	0.93586	0.76394053

Table 4 (Continued)

4	0.7628952	0.91448	0.697652413
5	0.7129862	0.89310	0.636767956
6	0.6663422	0.87172	0.580863842
7	0.6227497	0.85034	0.529549015
8	0.5820091	0.82896	0.482462266
9	0.5439337	0.80758	0.439270011
10	0.5083493	0.78620	0.399664212
11	0.4750928	0.76482	0.363360471
12	0.4440120	0.74344	0.330096250
13	0.4149644	0.72206	0.299629228
14	0.3878172	0.70068	0.271735783
15	0.3624460	0.67930	0.246209580
16	0.3387346	0.65792	0.222860266
17	0.3165744	0.63654	0.201512262
18	0.2958639	0.61516	0.182003646
19	0.2765083	0.59378	0.164185117
20	0.2584190	0.57240	0.147919036
21	0.2415131	0.55102	0.133078540
22	0.2257132	0.52964	0.119546720
23	0.2109469	0.50826	0.107215862
24	0.1971466	0.48688	0.095986746
25	0.1842492	0.46550	0.085767992
26	0.1721955	0.44412	0.076475462
27	0.1609304	0.42274	0.068031703
28	0.1504022	0.40136	0.060365432
29	0.1405628	0.37998	0.053411058
30	0.1313671	0.35860	0.047108248
31	0.1227730	0.33722	0.041401513
32	0.1147411	0.31584	0.036239837
33	0.1072347	0.29446	0.031576329
34	0.1002193	0.27308	0.027367898
35	0.0936629	0.25170	0.023574962
36	0.0875355	0.23032	0.020161166
37	0.0818088	0.20894	0.017093139
38	0.0764569	0.18756	0.014340248
39	0.0714550	0.16618	0.011874393
40	0.0667804	0.14480	0.009669799
41	0.0624116	0.12342	0.007702836
42	0.0583286	0.10204	0.005951847
43	0.0545127	0.08066	0.004396993
44	0.0509464	0.05928	0.003020105
45	0.0476135	0.03790	0.001804551
46	0.0444986	0.01652	0.000735117
	Total		10.81426900

(4) The final step was to determine the present value of the pension benefits and premiums. The present value of pension benefits was calculated for men and women who leave the pension fund. Furthermore, the present value of pension benefits for men and women, excluding pension fund insurance (caused by death, permanent disability, or early retirement), is that both men and women leave at the same age, namely 36 years.

Based on Eq. (21), with

$$v^{r-x} = v^{56-36} = v^{20} = (0.934579439)^{20} = 0.25841901, \text{ and}$$

$${}_{(56-36)}p_{36}^{(T)} = {}_{20}p_{36}^{(T)} = 1 - {}_{20}q_{36}^{(T)} = 1 - (20) \times q_{36}^{(T)}$$

$$= 1 - 20 \times (0.00274) = 1 - 0.04478 = 0.94522.$$

Then the present value of men's pension benefit was obtained as

$$\tilde{A}_x(a) = B_r p_x^{(T)} v^{r-x} \ddot{a}_r(a)$$

$$= \text{Rp } 58{,}553{,}387.62 \times 0.94522 \times 0.258419001 \times 10.80870923$$

$$\tilde{A}_{36}(a) = \text{Rp } 154{,}591{,}036.55.$$

whereas, for a woman, the present value of the pension benefit was obtained as

$$\tilde{A}_x(b) = B_r p_x^{(T)} v^{r-x} \ddot{a}_r(b)$$

$$= \text{Rp } 58{,}553{,}387.62 \times 0.94522 \times 0.258419001 \times 10.81426900$$

$$\tilde{A}_{36}(b) = \text{Rp } 154{,}617{,}723.54.$$

After calculating the present value of the benefit, it can determine the number of premium insurance of the pension fund group that the insurance participant of the pension fund would have to pay would be determined. Suppose the amount of pension funds provided by the company to the insured of the pension fund denoted by F was Rp 61,925,963. Based on Eq. (22), the men's total premium at the age x = 36 years, is as follows:

$$P_{36}(a) = \frac{B_{56} p_{36}^{(T)} v^{56-36} \ddot{a}_{56}(a) - F}{\ddot{a}_{56}(a)}$$

$$= \frac{\text{Rp } 154{,}591{,}036.55 - \text{Rp } 61{,}925{,}963}{10.80870923} = \text{Rp } 8{,}573{,}185{,}90.$$

whereas, the large premiums for women at the age of $x = 36$, are as follows:

$$P_{36}(b) = \frac{B_{56} p_{36}^{(T)} v^{56-36} \ddot{a}_{56}(b) - F}{\ddot{a}_{56}(b)}$$

$$= \frac{\text{Rp } 154{,}591{,}036.55 - \text{Rp } 61{,}925{,}963}{10.81426900} = \text{Rp } 8{,}571{,}246{,}06.$$

So, the total men's premium payable at the age, $x = 36$ was as large as Rp $8{,}573{,}185.55$. In contrast, the magnitude of women's premium payable was as large as Rp $8{,}571{,}246.06$ paid by the participants at the beginning of 2020.

Table 5: Present value of benefits and insurance premiums of male participant pension funds for three cases

x	$\tilde{A}_x(a)$(Rp)	$P_x(a)$(Rp)	x	$\tilde{A}_x(a)(Rp)$	$P_x(a)$(Rp)
23	63,031,058.68	102,241.23	40	200,943,940.88	12,861,663.21
24	67,692,970.72	533,551.93	41	214,637,460.32	14,128,560.04
25	72,716,564.21	998,324.68	42	228,999,619.08	15,457,318.03
26	78,078,519.01	1,494,401.94	43	244,651,443.93	16,905,393.33
27	83,776,140.14	2,021,534.36	44	261,675,615.93	18,480,435.42
28	89,858,392.60	2,584,252.11	45	279,704,437.49	20,148,425.67
29	96,324,638.06	3,182,496.11	46	297,898,023.39	21,831,659.58
30	102,946,846.01	3,795,169.44	47	319,602,140.29	23,839,680.74
31	110,255,718.63	4,471,371.61	48	342,937,653.32	25,998,635.39
32	118,095,866.03	5,196,726.25	49	368,363,191.81	28,350,954.98
33	126,414,644.35	5,966,362.86	50	396,197,612.57	30,926,139.51
34	135,310,814.26	6,789,418.58	51	426,596,021.82	33,738,539.09
35	144,796,346.78	7,667,000.94	52	459,790,659.21	36,809,640.05
36	154,591,036.55	8,573,185.90	53	496,442,303.77	40,200,576.36
37	165,028,280.66	9,538,818.69	54	537,072,292.02	43,959,581.01
38	176,199,237.80	10,572,333.14	55	582,110,588.68	48,126,433.47
39	188,135,565.85	11,676,658.16			

Table 6: Present value of benefits and insurance premiums of the pension fund of female participants for three cases

x	$\tilde{A}_x(b)$ (Rp)	$P_x(b)$(Rp)	x	$\tilde{A}_x(b)$(Rp)	$P_x(b)$(Rp)
23	63,041,939.71	103,194.84	40	200,978,629.75	12,858,258.54
24	67,704,656.53	534,358.22	41	214,674,513.10	14,124,722.63
25	72,729,117.24	998,972.21	42	229,039,151.20	15,453,026.76
26	78,091,997.68	1,494,880.02	43	244,693,678.01	16,900,607.43
27	83,790,602.38	2,021,832.39	44	261,720,788.89	18,475,111.53
28	89,873,904.82	2,584,357.93	45	279,752,722.76	20,142,532.03
29	96,341,266.54	3,182,397.58	46	297,949,449.41	21,825,191.00
30	102,964,617.68	3,794,861.65	47	319,657,313.08	23,832,526.27
31	110,274,752.03	4,470,832.84	48	342,996,854.52	25,990,743.48
32	118,116,252.88	5,195,939.72	49	368,426,782.21	28,342,259.58
33	126,436,467.26	5,965,313.44	50	396,266,008.01	30,916,564.50
34	135,334,172.91	6,788,088.03	51	426,669,664.93	33,728,003.43
35	144,821,342.92	7,665,370.62	52	459,870,032.69	36,798,055.39
36	154,617,723.54	8,571,246.06	53	496,528,004.42	40,187,833.45
37	165,056,769.43	9,536,549.02	54	537,165,006.60	43,945,554.12
38	176,229,655.01	10,569,710.45	55	582,211,078.20	48,110,983.29
39	188,168,043.62	11,673,658.26			

The table above shows that the present value of the benefit earned for men and women with the same salary, level of salary increase, and age of entry, was found to be different, and men's premium payable was greater than women's.

4. Conclusion

Based on the results, for cases (death, permanent disability, and early retirement) the value of the benefits and premiums that must be paid is currently not the same for male and female participants. Male participants received lower pension benefits than female participants. Men paid a higher premium (Rp 48,126,433.47) than women (Rp 48,110,983.29). The reason is that men's initial lifetime annuity is smaller than women's because the men's maximum age is lower than women's. Calculating the withdrawal probability using the uniform assumption also affects the annuity.

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