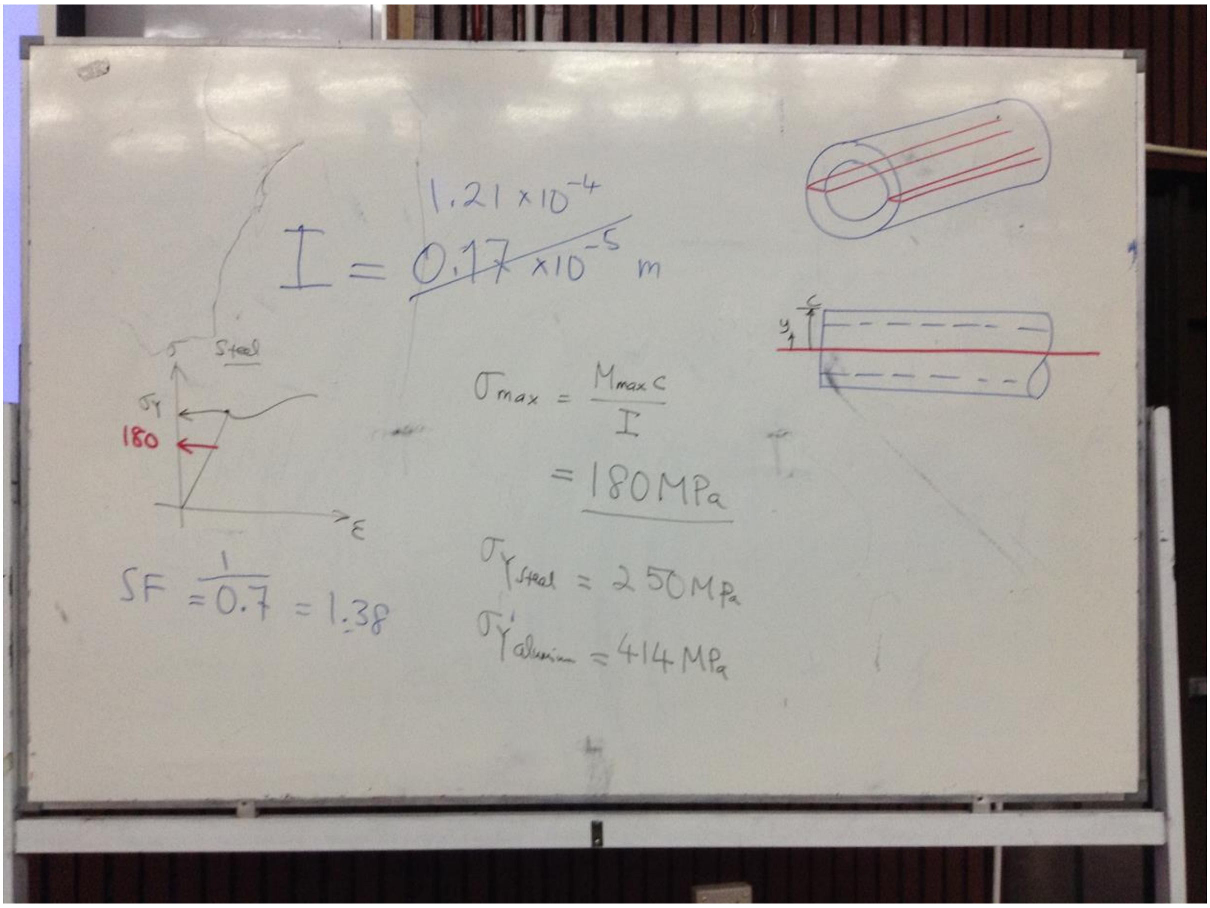
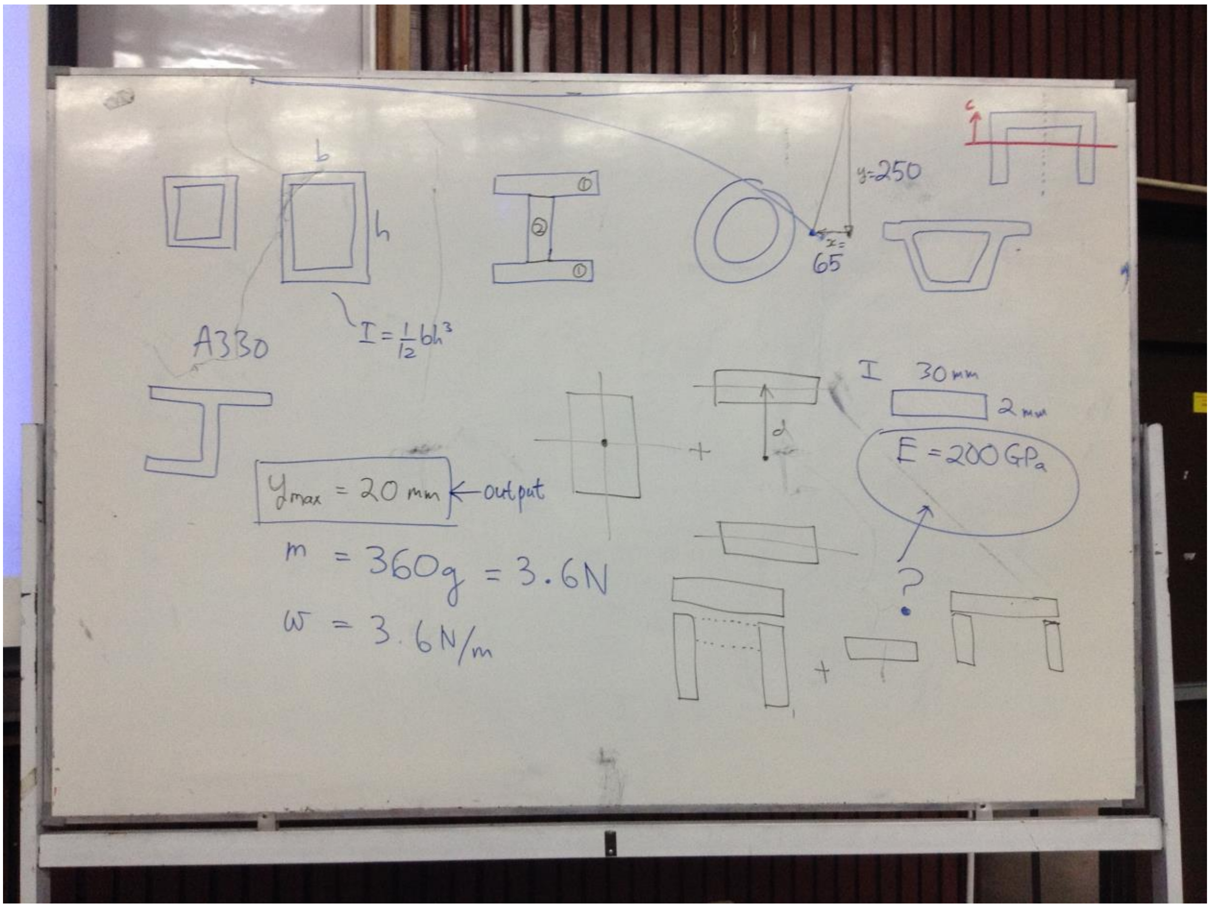


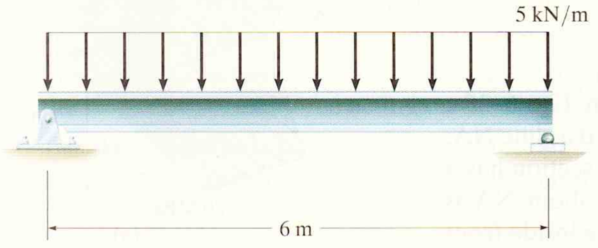
1.21 x10^-4
I = 0.17 x10^-5 m
Steel
σY
180
ε
σmax = Mmax c / I
= 180MPa
σY steel = 250MPa
σY aluminium = 414MPa
SF = 1/0.7 = 1.38
y
c



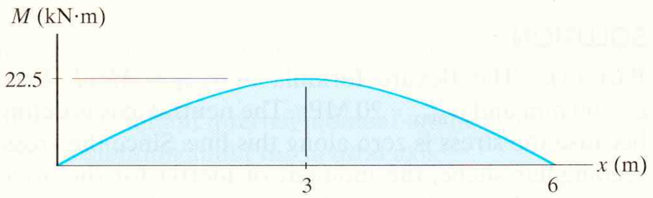
b
h
I = 1/12 bh^3
y=250
x=
65
c
A330
ymax = 20 mm ← output
m = 360g = 3.6N
w = 3.6N/m
I 30mm
2mm
E = 200GPa
d
?

EXAMPLE 6.12

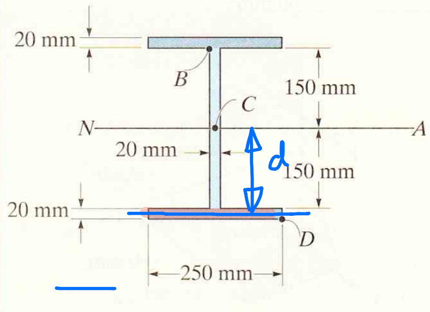
The simply supported beam in Fig. 6–26*a* has the cross-sectional area shown in Fig. 6–26*b*. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.



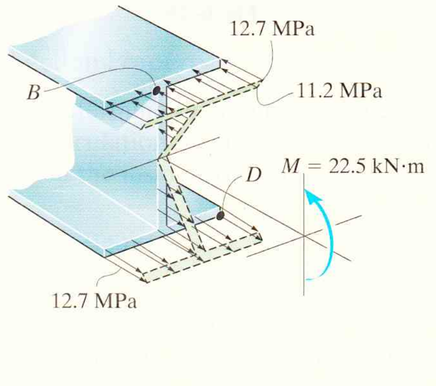
(a)



(c)



(b)



(d)

Fig. 6–26

SOLUTION

Maximum Internal Moment. The maximum internal moment in the beam, $M = 22.5\ \text{kN}\cdot\text{m}$, occurs at the center.

Section Property. By reasons of symmetry, the neutral axis passes through the centroid C at the midheight of the beam, Fig. 6–26*b*. The area is subdivided into the three parts shown, and the moment of inertia of each part is calculated about the neutral axis using the parallel-axis theorem. (See Eq. A–5 of Appendix A.) Choosing to work in meters, we have

$$I = \Sigma(\bar{I} + Ad^2)$$

$$= 2\left[\frac{1}{12}(0.25\ \text{m})(0.020\ \text{m})^3 + \boxed{(0.25\ \text{m})(0.020\ \text{m})(0.160\ \text{m})^2}\right]$$

$$+ \left[\frac{1}{12}(0.020\ \text{m})(0.300\ \text{m})^3\right]$$

$$= 301.3(10^{-6})\ \text{m}^4$$

$$\sigma_{\max} = \frac{Mc}{I}; \qquad \sigma_{\max} = \frac{22.5(10^3)\ \text{N}\cdot\text{m}(0.170\ \text{m})}{301.3(10^{-6})\ \text{m}^4} = 12.7\ \text{MPa} \qquad \textit{Ans.}$$

A three-dimensional view of the stress distribution is shown in Fig. 6–26*d*. Notice how the stress at points B and D on the cross section develops a force that contributes a moment about the neutral axis that has the same direction as **M**. Specifically, at point B, $y_B = 150$ mm, and so

$$\sigma_B = -\frac{My_B}{I}; \qquad \sigma_B = -\frac{22.5(10^3)\ \text{N}\cdot\text{m}(0.150\ \text{m})}{301.3(10^{-6})\ \text{m}^4} = -11.2\ \text{MPa}$$

EXAMPLE 6.13

The beam shown in Fig. 6–27*a* has a cross-sectional area in the shape of a channel, Fig. 6–27*b*. Determine the maximum bending stress that occurs in the beam at section *a–a*.

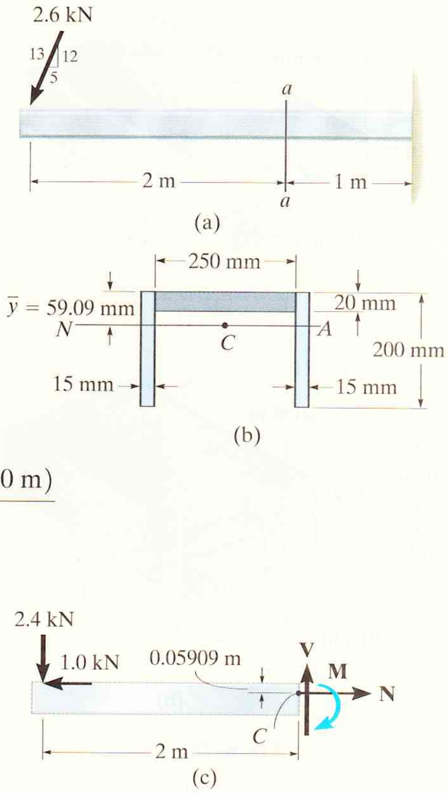


Fig. 6–27

SOLUTION

Internal Moment. Here the beam's support reactions do not have to be determined. Instead, by the method of sections, the segment to the left of section *a–a* can be used, Fig. 6–27*c*. In particular, note that the resultant internal axial force **N** passes through the centroid of the cross section. Also, realize that *the resultant internal moment must be calculated about the beam's neutral axis* at section *a–a*.

To find the location of the neutral axis, the cross-sectional area is subdivided into three composite parts as shown in Fig. 6–27*b*. Using Eq. A–2 of Appendix A, we have

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[0.100\text{ m}](0.200\text{ m})(0.015\text{ m}) + [0.010\text{ m}](0.02\text{ m})(0.250\text{ m})}{2(0.200\text{ m})(0.015\text{ m}) + 0.020\text{ m}(0.250\text{ m})}$$

$$= 0.05909\text{ m} = 59.09\text{ mm}$$

This dimension is shown in Fig. 6–27*c*.

Applying the moment equation of equilibrium about the neutral axis, we have

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad 2.4\text{ kN}(2\text{ m}) + 1.0\text{ kN}(0.05909\text{ m}) - M = 0$$

$$M = 4.859\text{ kN}\cdot\text{m}$$

Section Property. The moment of inertia about the neutral axis is determined using the parallel-axis theorem applied to each of the three composite parts of the cross-sectional area. Working in meters, we have

$$I = \left[\frac{1}{12}(0.250\text{ m})(0.020\text{ m})^3 + (0.250\text{ m})(0.020\text{ m})(0.05909\text{ m} - 0.010\text{ m})^2\right]$$

$$+ 2\left[\frac{1}{12}(0.015\text{ m})(0.200\text{ m})^3 + (0.015\text{ m})(0.200\text{ m})(0.100\text{ m} - 0.05909\text{ m})^2\right]$$

$$= 42.26(10^{-6})\text{ m}^4$$

Maximum Bending Stress. The maximum bending stress occurs at points farthest away from the neutral axis. This is at the bottom of the beam, $c = 0.200\text{ m} - 0.05909\text{ m} = 0.1409\text{ m}$. Thus,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{4.859(10^3)\text{ N}\cdot\text{m}(0.1409\text{ m})}{42.26(10^{-6})\text{ m}^4} = 16.2\text{ MPa} \quad \textit{Ans.}$$

Show that at the top of the beam the bending stress is $\sigma' = 6.79$ MPa.

NOTE: The normal force of $N = 1$ kN and shear force $V = 2.4$ kN will also contribute additional stress on the cross section. The superposition of all these effects will be discussed in Chapter 8.

EXAMPLE 7.1

The solid shaft and tube shown in Fig. 7–9*a* are subjected to the shear force of 4 kN. Determine the shear stress acting over the diameter of each cross section.

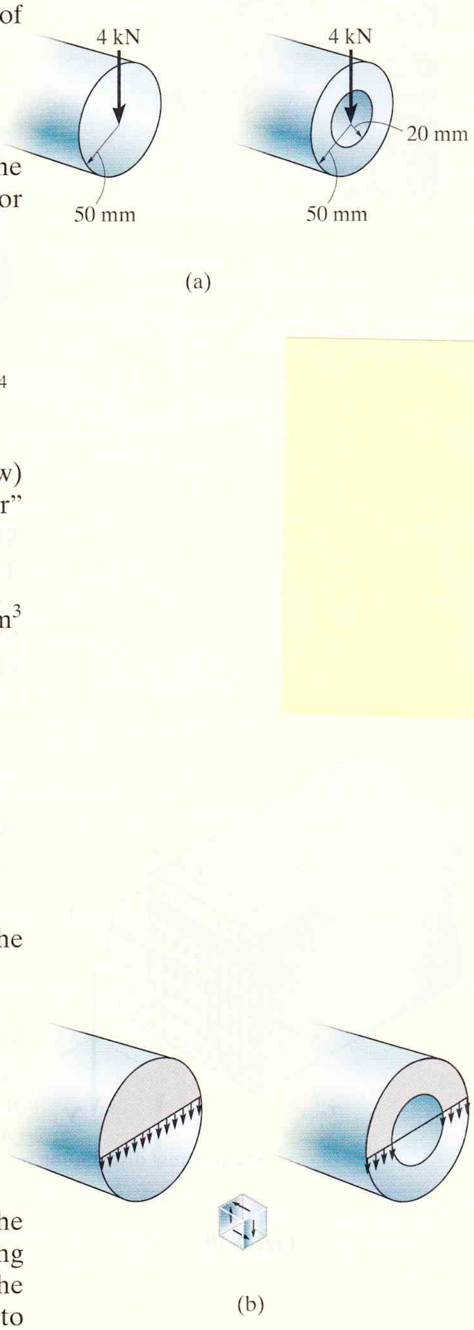


Fig. 7–9

SOLUTION

Section Properties. Using the table on the inside front cover, the moment of inertia of each section, calculated about its diameter (or neutral axis), is

$$I_{\text{solid}} = \frac{1}{4}\pi c^4 = \frac{1}{4}\pi(0.05\text{ m})^4 = 4.909(10^{-6})\text{ m}^4$$

$$I_{\text{tube}} = \frac{1}{4}\pi(c_o^4 - c_i^4) = \frac{1}{4}\pi[(0.05\text{ m})^4 - (0.02\text{ m})^4] = 4.783(10^{-6})\text{ m}^4$$

The semicircular area shown shaded in Fig. 7–9*b*, above (or below) each diameter, represents Q, because this area is "held onto the member" by the longitudinal shear stress along the diameter.

$$Q_{\text{solid}} = \bar{y}'A' = \frac{4c}{3\pi}\left(\frac{\pi c^2}{2}\right) = \frac{4(0.05\text{ m})}{3\pi}\left(\frac{\pi(0.05\text{ m})^2}{2}\right) = 83.33(10^{-6})\text{ m}^3$$

$$Q_{\text{tube}} = \Sigma\bar{y}'A' = \frac{4c_o}{3\pi}\left(\frac{\pi c_o^2}{2}\right) - \frac{4c_i}{3\pi}\left(\frac{\pi c_i^2}{2}\right)$$

$$= \frac{4(0.05\text{ m})}{3\pi}\left(\frac{\pi(0.05\text{ m})^2}{2}\right) - \frac{4(0.02\text{ m})}{3\pi}\left(\frac{\pi(0.02\text{ m})^2}{2}\right)$$

$$= 78.0(10^{-6})\text{ m}^3$$

Shear Stress. Applying the shear formula where $t = 0.1$ m for the solid section, and $t = 2(0.03\text{ m}) = 0.06$ m for the tube, we have

$$\tau_{\text{solid}} = \frac{VQ}{It} = \frac{4(10^3)\text{ N}(83.33(10^{-6})\text{ m}^3)}{4.909(10^{-6})\text{ m}^4(0.1\text{ m})} = 679\text{ kPa} \qquad \textit{Ans.}$$

$$\tau_{\text{tube}} = \frac{VQ}{It} = \frac{4(10^3)\text{ N}(78.0(10^{-6})\text{ m}^3)}{4.783(10^{-6})\text{ m}^4(0.06\text{ m})} = 1.09\text{ MPa} \qquad \textit{Ans.}$$

NOTE: As discussed in the limitations for the shear formula, the calculations performed here are valid since the shear stress along the diameter is vertical and therefore tangent to the boundary of the cross section. An element of material on the diameter is subjected to "pure shear" as shown in Fig. 7–9*b*.